The muon g-2 and the bounds on the Higgs mass

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June 24 2008

The present experimental values:

$$a_e = 1159652180.73\ (28) \times 10^{-12}$$

0.24 parts per <u>billion</u> !! Hanneke et al., PRL100 (2008) 120801

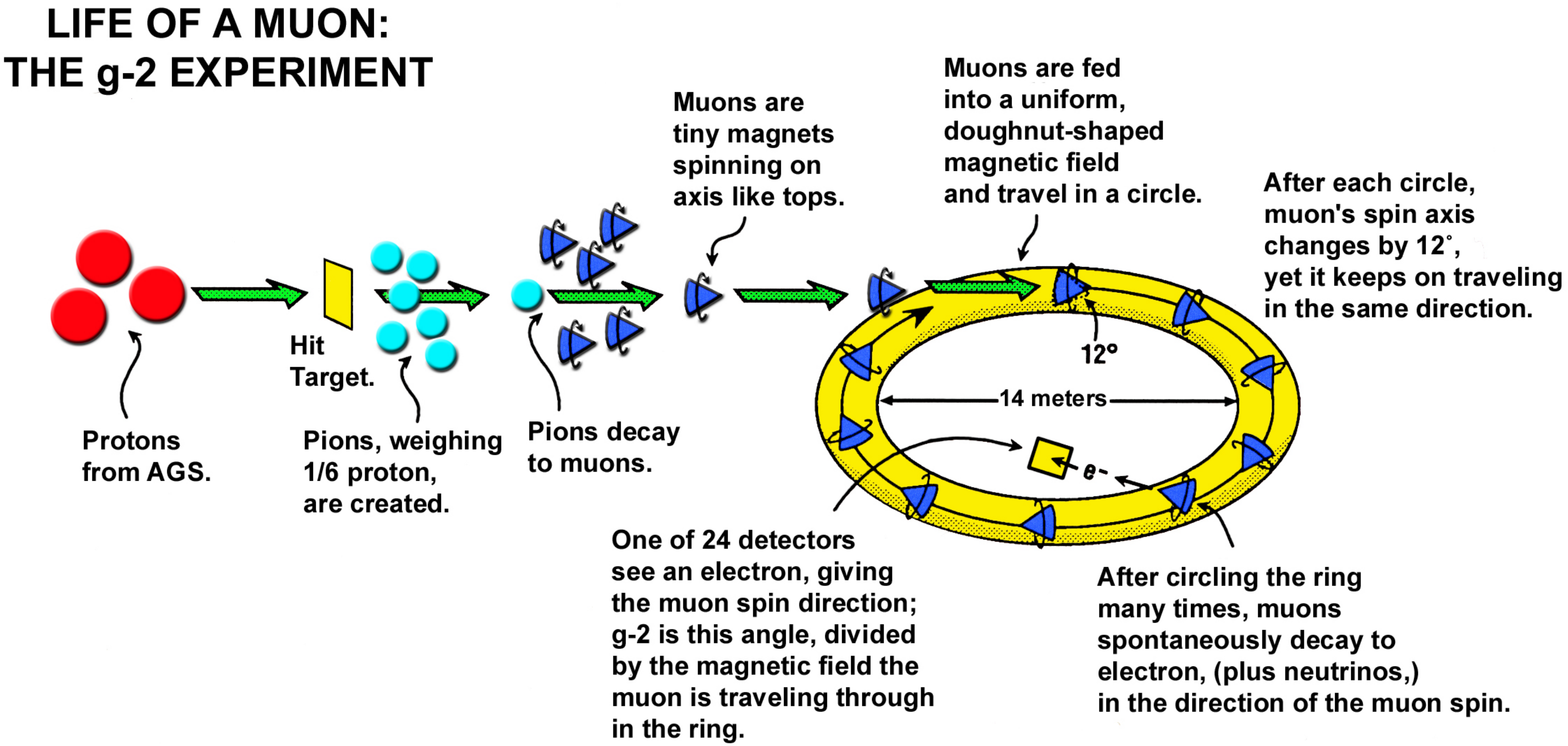
$$a_\mu = 116592080\ (63) \times 10^{-11}$$

0.5 parts per <u>million</u> !! E821 – Final Report: PRD73 (2006) 072003

$$a_\tau = -0.018\ (17)$$

DELPHI - EPJC35 (2004) 159 [$a_\tau^{SM} = 117721(5) \times 10^{-8}$, Eidelman & MP '07]

The experiment



LIFE OF A MUON: THE g-2 EXPERIMENT

Protons from AGS.

Hit Target.

Pions, weighing 1/6 proton, are created.

Pions decay to muons.

Muons are tiny magnets spinning on axis like tops.

Muons are fed into a uniform, doughnut-shaped magnetic field and travel in a circle.

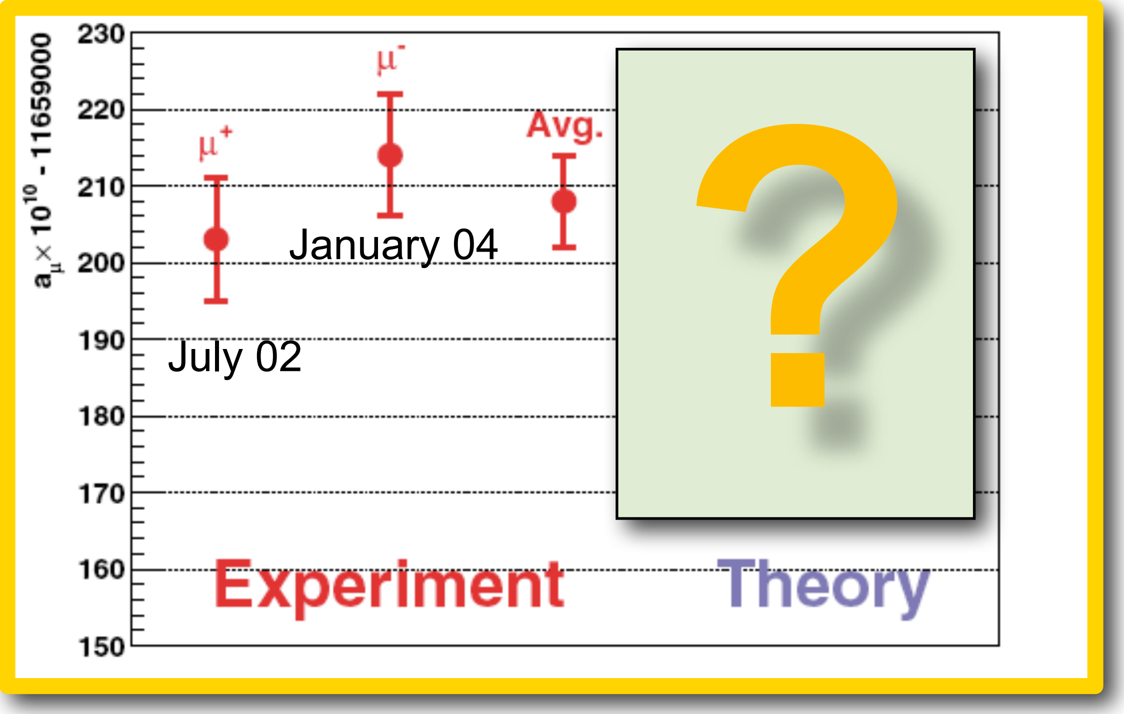
After each circle, muon's spin axis changes by 12˚, yet it keeps on traveling in the same direction.

One of 24 detectors see an electron, giving the muon spin direction; g-2 is this angle, divided by the magnetic field the muon is traveling through in the ring.

After circling the ring many times, muons spontaneously decay to electron, (plus neutrinos,) in the direction of the muon spin.

Homepage of E821

The experimental result



- Today: $a_\mu^{EXP} = (116592080 \pm 54_{stat} \pm 33_{sys}) \times 10^{-11}$ [0.5ppm].
- Future: a new $(g-2)_\mu$ exp? E969 aims at 0.2 ppm ($25 \text{x} 10^{-11}$).

  A "conservative" upgrade could reach 0.25 ppm.

  A "Legacy" exp. aims at 0.14 ppm [D. Hertzog, Glasgow Oct '07].
- Are theorists ready for this? [no]

The anomalous magnetic moment: the basics

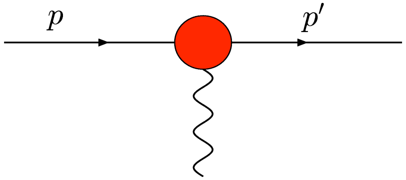
- The Dirac theory predicts for a lepton l=e,μ,τ:

$$\vec{\mu}_l = g_l \left(\frac{e}{2m_l c}\right) \vec{s} \qquad g_l = 2$$

- QFT predicts deviations from the Dirac value:

$$g_l = 2\,(1 + a_l)$$

- Study the photon-lepton vertex:



$$\bar{u}(p')\Gamma_\mu u(p) = \bar{u}(p')\left[\gamma_\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2m}F_2(q^2) + \ldots\right]u(p)$$

$$\boxed{F_1(0) = 1 \qquad F_2(0) = a_l}$$

The QED contribution to a_μ

$a_\mu^{QED} = (1/2)(\alpha/\pi)$ Schwinger 1948

$+ \ 0.765857410\ (27)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '04

$+ \ 24.05050964\ (43)\ (\alpha/\pi)^3$

Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek; MP '04; Friot, Greynat, de Rafael '05

$+ \ 130.805\ (8)\ (\alpha/\pi)^4$ Revised!

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05; Aoyama, Hayakawa, Kinoshita & Nio, June & Dec 2007

$+ \ 663\ (20)\ (\alpha/\pi)^5$ In progress

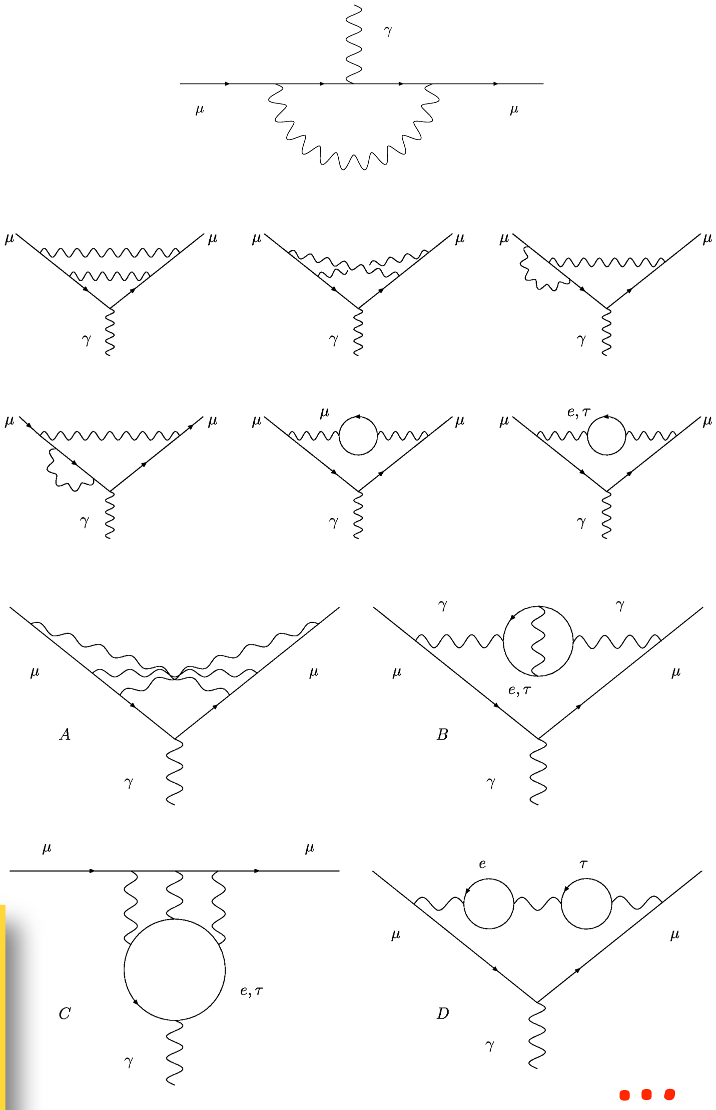
Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta, Karshenboim,…, Kataev, Kinoshita & Nio March '06.

Adding up, I get:

$a_\mu^{QED} = 116584718.09\ (14)(04) \times 10^{-11}$

mainly from 5-loop unc ↵ ↳ from new $\delta\alpha$('08)

with $\alpha = 1/137.035999084(51)$ [0.37 ppb]



[A parenthesis on the electron g-2…

a_e^{SM} = $(1/2)(\alpha/\pi)$ - $0.328\,478\,444\,002\,90(60)\,(\alpha/\pi)^2$

Schwinger 1948 Sommerfield; Petermann; Suura & Wichmann '57; Elend '66; MP '06

$A_2^{(4)}\,(m_e/m_\mu) = 5.197\,386\,70\,(28) \times 10^{-7}$

$A_2^{(4)}\,(m_e/m_\tau) = 1.837\,62\,(60) \times 10^{-9}$

$+\ 1.181\,234\,016\,827\,(19)\,(\alpha/\pi)^3$

Kinoshita, Barbieri, Laporta, Remiddi, … , Li, Samuel; Mohr & Taylor '05; MP '06

$A_2^{(6)}\,(m_e/m_\mu) = -7.373\,941\,64\,(29) \times 10^{-6}$

$A_2^{(6)}\,(m_e/m_\tau) = -6.5819\,(19) \times 10^{-8}$

$A_3^{(6)}\,(m_e/m_\mu, m_e/m_\tau) = 1.909\,45\,(62) \times 10^{-13}$

$-\ \cancel{1.7283}\,(35)\,(\alpha/\pi)^4$ Revised value: -1.9144 (35)

Kinoshita & Lindquist '81, … , Kinoshita & Nio '05; Aoyama, Hayakawa, Kinoshita & Nio, June '07

$+\ 0.0\,(4.6)\,(\alpha/\pi)^5$ In progress (12672 mass ind. diagrams!)

Mohr & Taylor '05; Aoyama, Hayakawa, Kinoshita, Nio & Watanabe, June 2008 (more in progress).

$+\ 1.682\,(20) \times 10^{-12}$ Hadronic

Mohr, Taylor & Newell '08; Davier & Hoecker '98, Krause '97, Knecht '03

$+\ 0.0297\,(5) \times 10^{-12}$ Electroweak

Mohr & Taylor '05; Czarnecki, Krause, Marciano '96

... and the best determination of alpha]

- The new measurement of the electron g-2 is:

  $a_e^{exp} = 1159652180.73\ (28) \times 10^{-12}$ Hanneke et al, PRL100 (2008) 120801

  vs. old (factor of 15 improvement, 1.8σ difference):

  $a_e^{exp} = 1159652188.3\ (4.2) \times 10^{-12}$ Van Dyck et al, PRL59 (1987) 26

- Equating $a_e^{SM}(\alpha) = a_e^{exp}$ → best determination of alpha to date:

$$\boxed{\alpha^{-1} = 137.035\,999\,084\ \underbrace{(12)}_{\delta C_4^{qed}}\underbrace{(37)}_{\delta C_5^{qed}}\underbrace{(2)}_{\delta a_e^{had}}\underbrace{(33)}_{\delta a_e^{exp}}\ [0.37\text{ppb}]}$$

Hanneke et al, '08

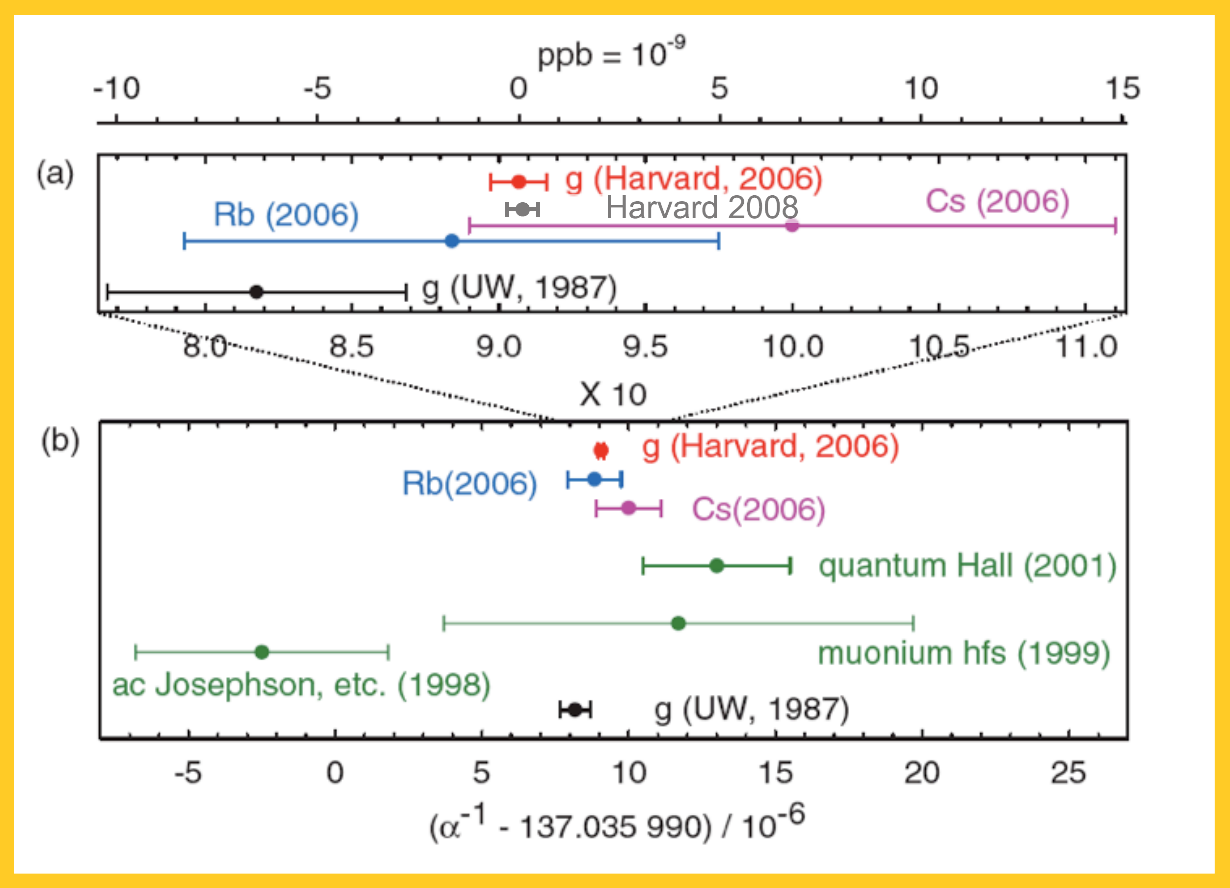
(smaller than th!)

- Compare it with other determinations (independent of a_e):

  $\alpha^{-1} = 137.036\,000\,00\ (110)\ [8.0\text{ ppb}]$ PRA73 (2006) 032504 (Cs)
  $\alpha^{-1} = 137.035\,998\,78\ (91)\ [6.7\text{ ppb}]$ PRL96 (2006) 033001 (Rb)

$\Delta = +0.8$ and $-0.3\ \sigma$ → beautiful test of QED at 4-loop level!

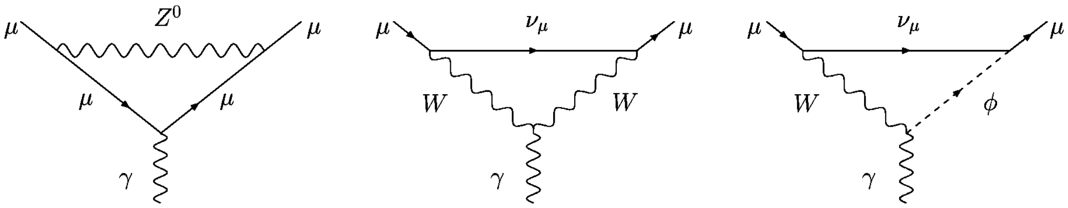
Old and new determinations of alpha



Gabrielse, Hanneke, Kinoshita, Nio & Odom, PRL99 (2007) 039902
Hanneke, Fogwell & Gabrielse, PRL100 (2008) 120801

The Electroweak contribution

- One-loop term:



$$a_\mu^{\mathrm{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2+O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right]\approx 195\times10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda.

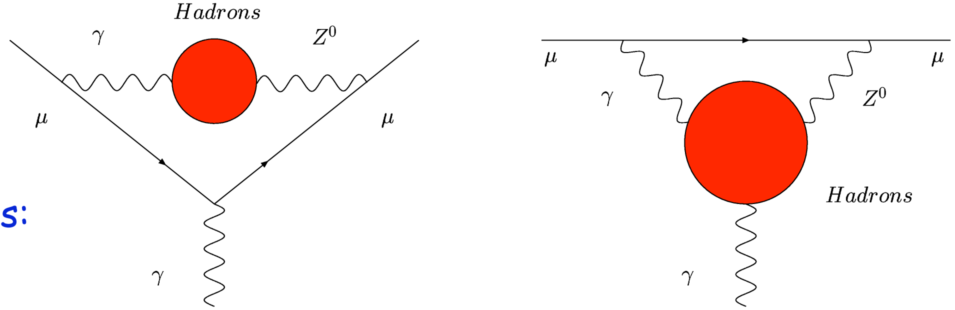
- One-loop plus higher-order terms:

a_μ^{EW} = 154 (2) (1) × 10^{-11}

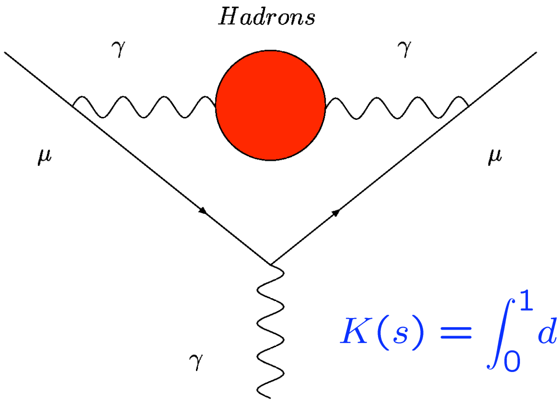
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano, Vainshtein '02; Degrassi, Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk, Czarnecki '05; Vainshtein '03.

Higgs mass, M_{top} error,
3-loop nonleading logs

Hadronic loop uncertainties:



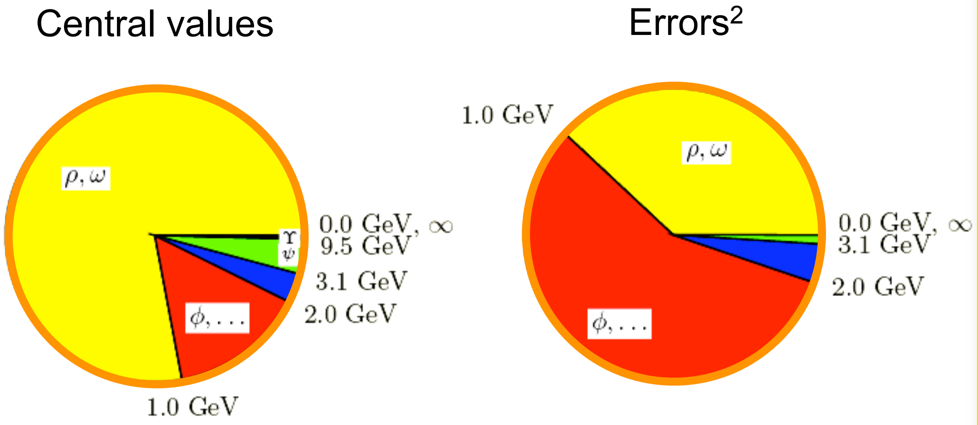
The hadronic leading-order (HLO) contribution



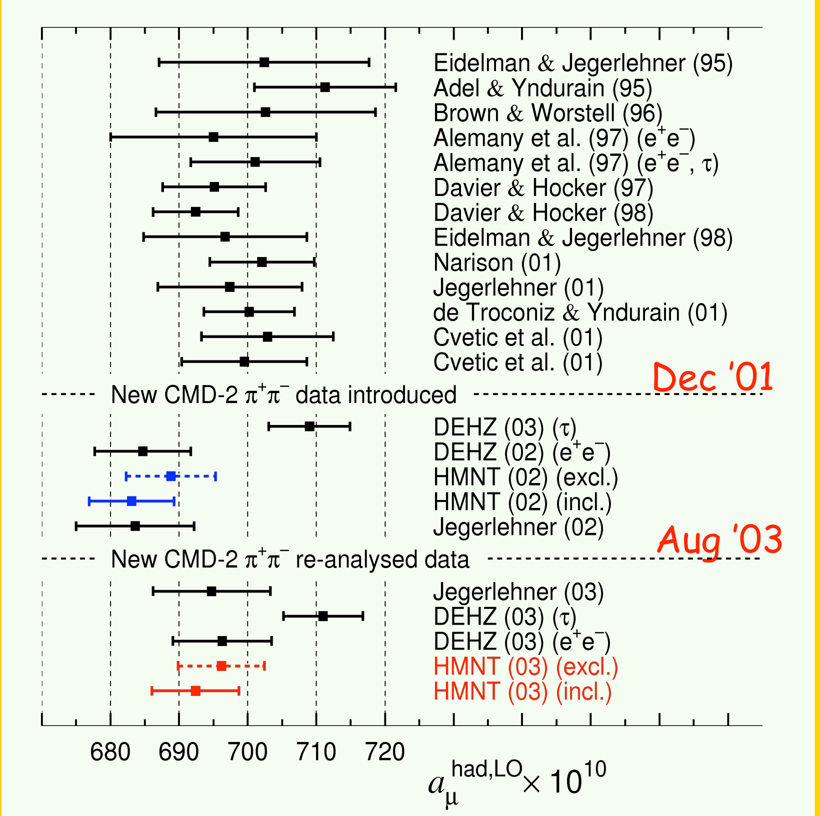
$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}(s) = \frac{\alpha^2}{3\pi^2}\int_{4m_\pi^2}^{\infty} \frac{ds}{s} K(s)\, R(s)$$

$$K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (1-x)s/m_\mu^2}$$

Bouchiat & Michel 1961; Gourdin & de Rafael 1969



F. Jegerlehner, PhiPsi 08, Frascati, April 2008



Hagiwara et al., PRD 69 (2004) 093003

The HLO contribution: e^+e^- data

$a_\mu^{HLO} = 6909\,(39)_{exp}\,(19)_{rad}\,(7)_{qcd} \times 10^{-11}$ S. Eidelman, ICHEP06; M. Davier, TAU06

$= 6894\,(42)_{exp}\,(18)_{rad} \times 10^{-11}$ Hagiwara, Martin, Nomura, Teubner, PLB649(2007)173

$= 6923\,(60)_{tot} \times 10^{-11}$ F. Jegerlehner, PhiPsi 08, Frascati, April 2008

$= 6944\,(48)_{exp}\,(10)_{rad} \times 10^{-11}$ de Troconiz & Yndurain, PRD71 (2005) 73008

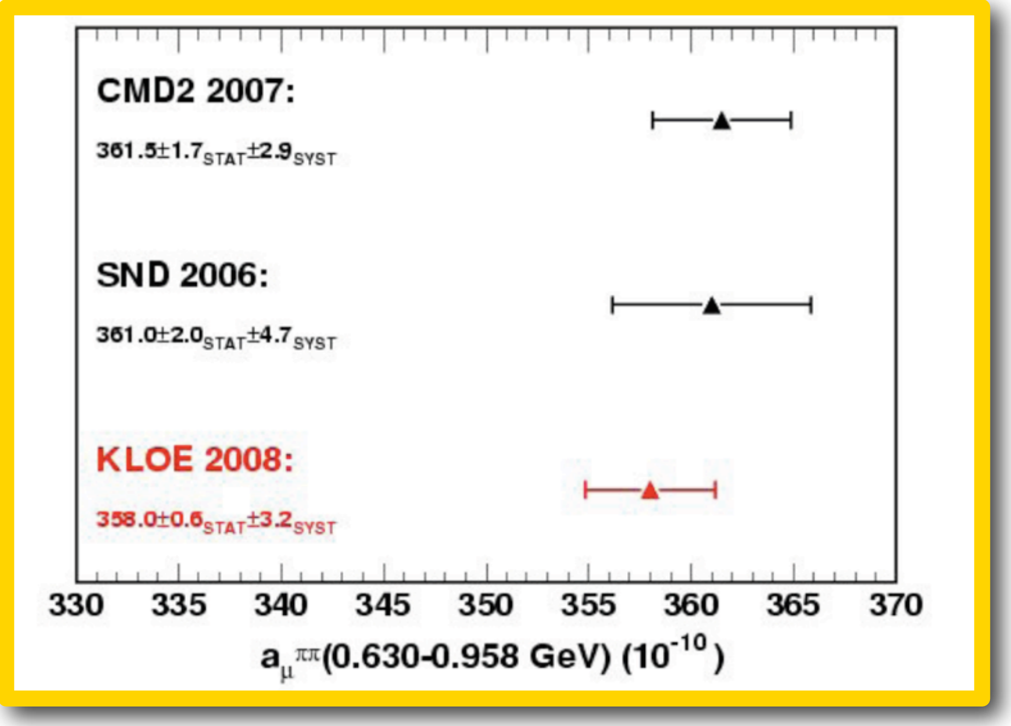
- Radiative Corrections (Luminosity, ISR, Vacuum Polarization, **FSR**) are a very delicate issue! Are they all under control?
- CMD2's 1998 $\pi^+\pi^-$ data in the ρ energy range, published in 2007, agree well with their earlier (1995) ones.
- SND's $\pi^+\pi^-$ 2006 data reanalysis appears to be in good agreement with CMD2.

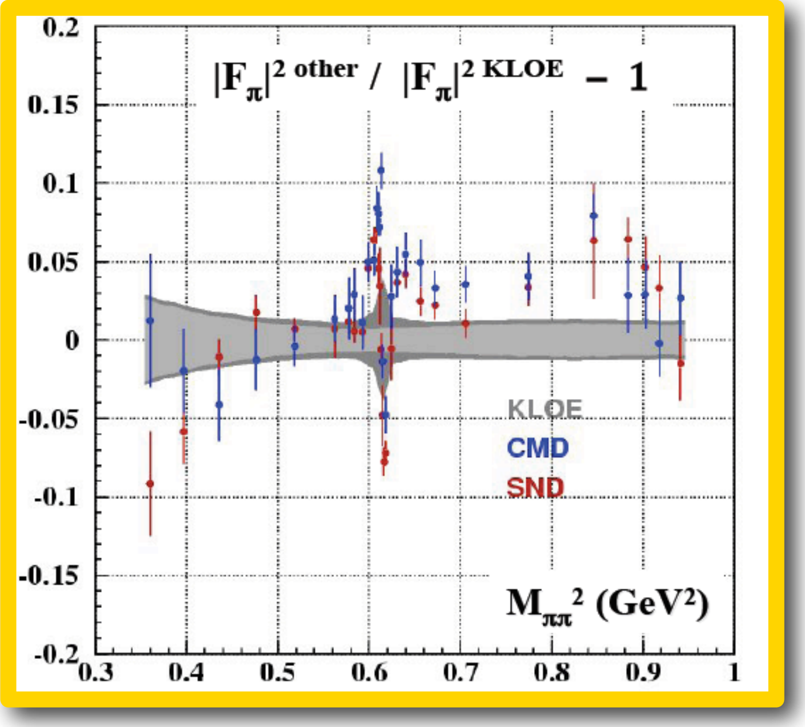
The HLO contribution: e^+e^- data (ISR Method)

- The RADIATIVE RETURN (ISR) Method: KLOE & BABAR. Collider operates at fixed energy but s_π can vary continuously. Important independent method made possible by beautiful interplay between theory and experiment.
- Discrepancies between KLOE's (2001) and CMD2's results even if their contributions to a_μ^{HLO} are similar (see table).
- Comparison in the range $s_\pi \in [0.37, 0.93]$ GeV^2:

$a_\mu^{\pi\pi} = (3786 \pm 27_{stat} \pm 23_{sys+th}) \times 10^{-11}$	CMD2 (95)	PLB578 (2004) 285
$a_\mu^{\pi\pi} = (3771 \pm 19_{stat} \pm 27_{sys+th}) \times 10^{-11}$	CMD2 (95+98)	S.Eidelman, ICHEP '06
$a_\mu^{\pi\pi} = (3756 \pm 8_{stat} \pm 48_{sys+th}) \times 10^{-11}$	KLOE	G.Venanzoni, ICHEP '04
$a_\mu^{\pi\pi} = (3768 \pm 13_{stat} \pm 47_{sys+th}) \times 10^{-11}$	SND (revised)	S.Eidelman, ICHEP '06

- PhiPsi08: KLOE presented an update of its 2001 data analysis (some differences in $a_\mu^{\pi\pi}$ w.r.t. published value) & the new 2002 data analysis. Final results coming soon...





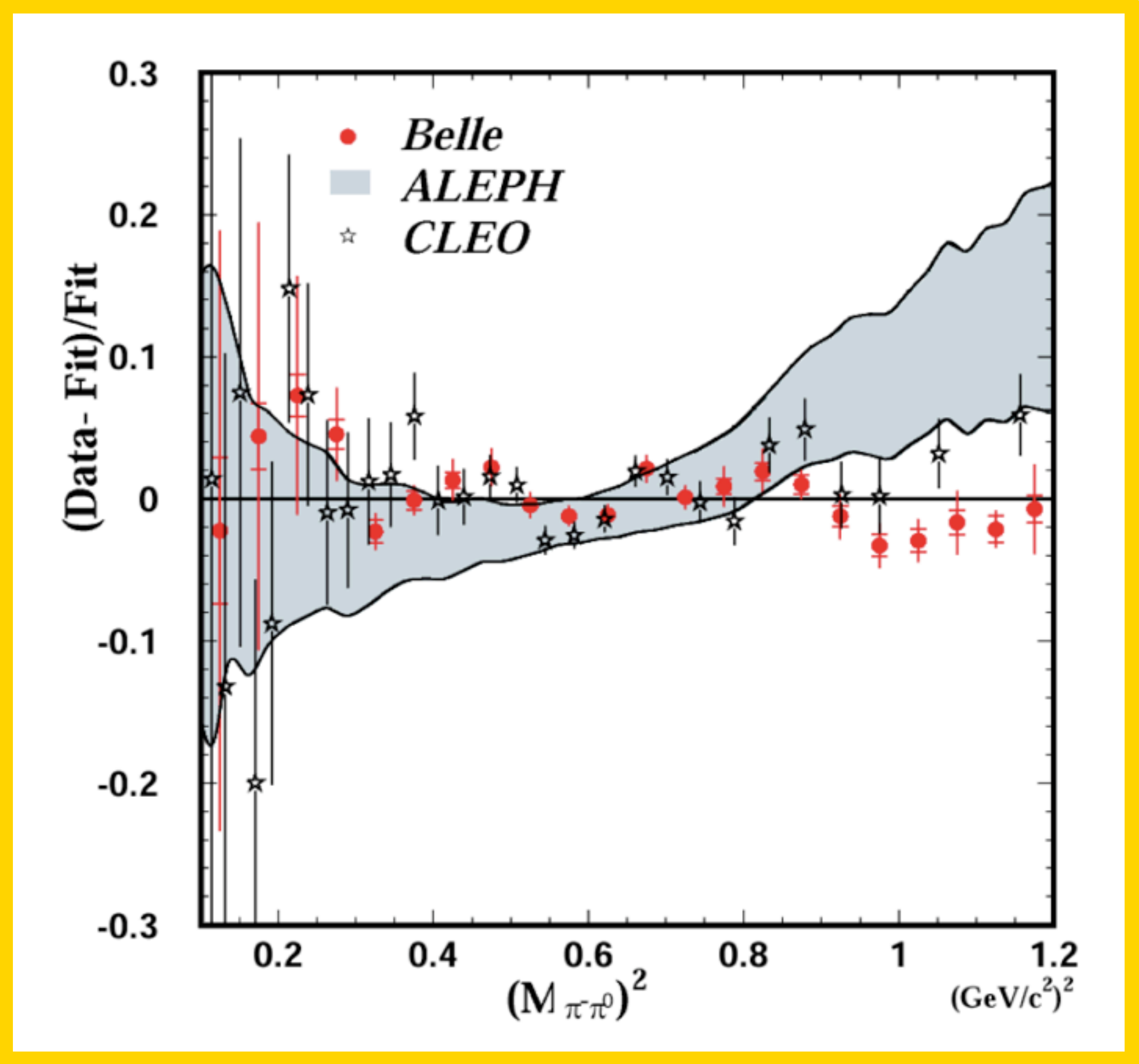
F. Nguyen, PhiPsi'08, Frascati, April 2008

The HLO contribution: Tau-decay data

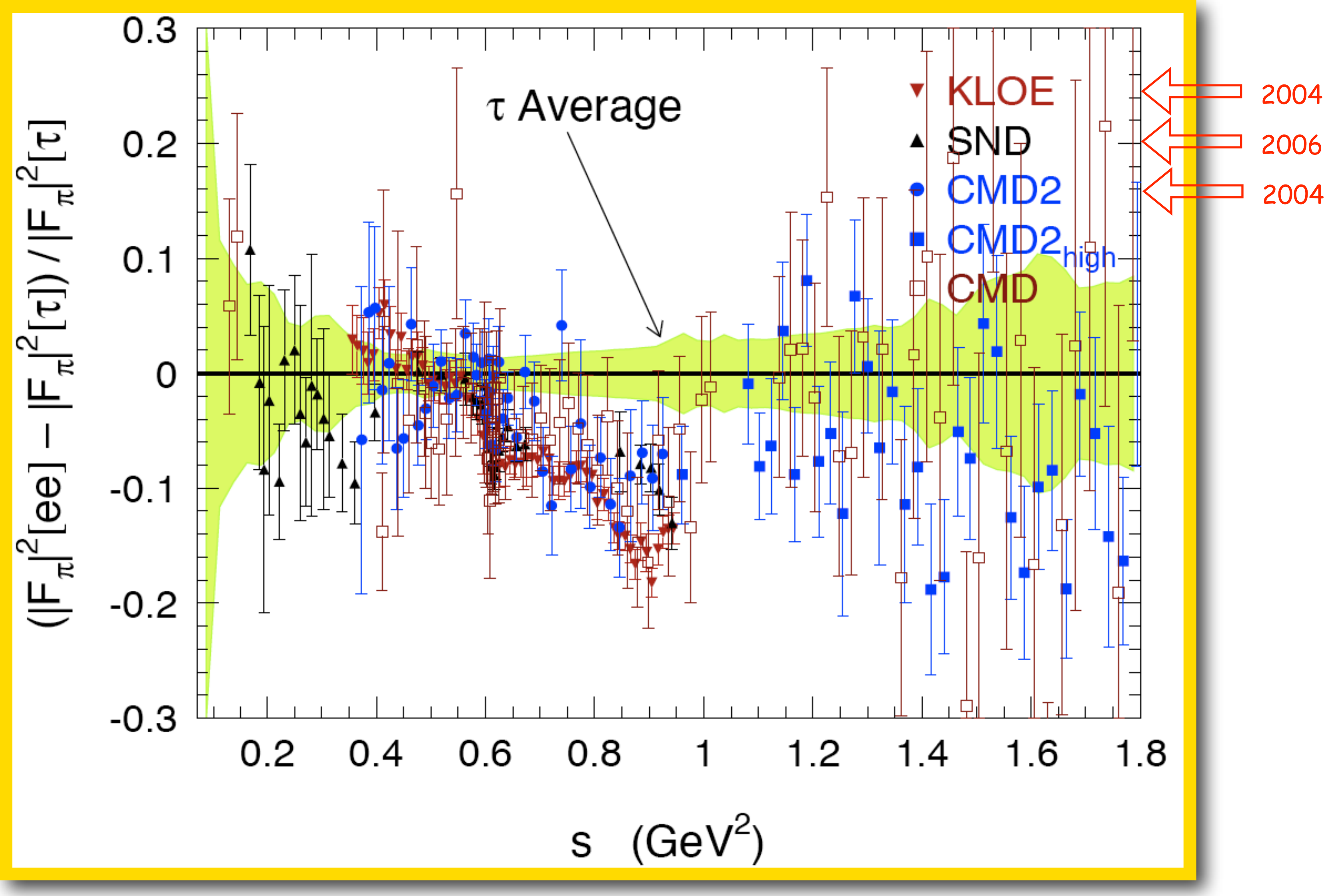
- TAU DATA: Many data sets (Aleph, Cleo, Opal & now Belle).
- The tau data of ALEPH and CLEO are significantly higher than CMD2 e^+e^- ones above ~ 0.85 GeV. KLOE confirms this discrepancy with the tau data.
- The recent $a_\mu^{\pi\pi}$ tau result of BELLE (arXiv:0805.3773) is in agreement with the previous Aleph-Cleo-Opal one, even if deviations from ALEPH's spectral functions are observed.
- Latest value, still (Davier, Eidelman, Hoecker, Zhang, EPJC31 (2003) 503):

$$\boxed{a_\mu^{HLO} = 7110\ (58) \times 10^{-11}}$$

- Inconsistencies in the e^+e^- or tau data? Are all possible isospin-breaking (IB) effects taken into account? Recent additional IB corrections somewhat reduce the diff. with e^+e^- data. Also, recent claims that e^+e^- and tau data are consistent after IB effects & vector meson mixings are considered (Marciano&Sirlin '88; Cirigliano, Ecker, Neufeld '01-'02, Flores-Baez et al. '06 &' 07, Benayoun et al.'07, Davier@Glasgow g-2 workshop, Oct '07).



Fujikawa, Hayashii, Eidelman [for the Belle Collab.], arXiv:0805.3773, May '08



2004

2006

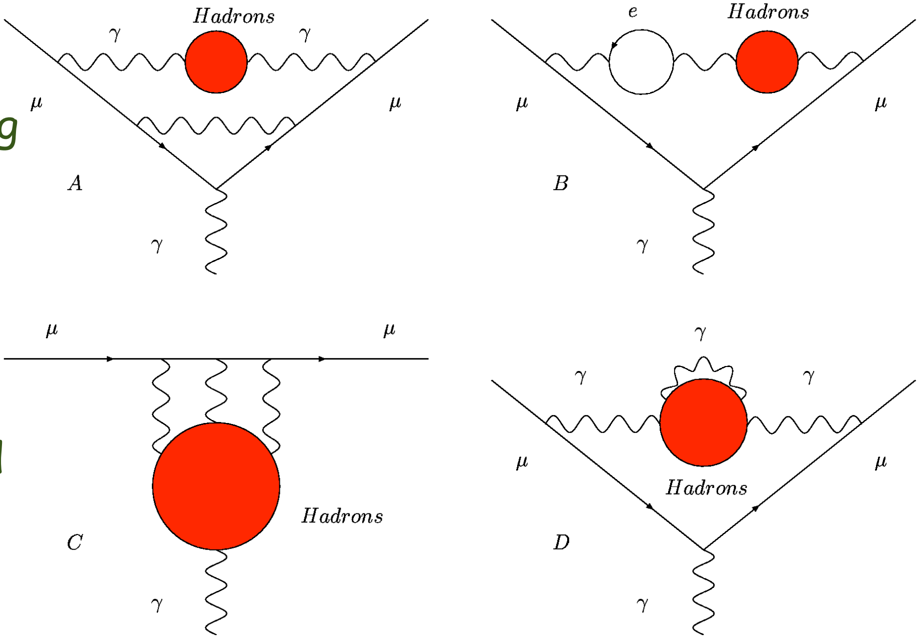
2004

M. Davier at TAU 06, Pisa, September 2006

The hadronic higher-order (HHO) contributions

- ## Vacuum Polarization

$O(\alpha^3)$ contributions of diagrams containing hadronic vacuum polarization insertions:



$$\boxed{a_\mu^{\text{HHO}}(\text{vp}) = -98\ (1) \times 10^{-11}}$$

Krause '96, Alemany et al. '98, Hagiwara et al. '03 & '06

Shifts by ~ -3×10^{-11} if tau data are used instead of the e^+e^- ones Davier & Marciano '04

- ## Light-by-Light

The contribution of the hadronic l-b-l diagrams had a troubled life. The latest values vary between:

$a_\mu^{\text{HHO}}(\text{lbl}) = +80\ (40) \times 10^{-11}$	Knecht & Nyffeler '02
$a_\mu^{\text{HHO}}(\text{lbl}) = +136\ (25) \times 10^{-11}$	Melnikov & Vainshtein '03
$a_\mu^{\text{HHO}}(\text{lbl}) = +110\ (40) \times 10^{-11}$	Bijnens & Prades '07

based also on Hayakawa, Kinoshita '98 & '02; Bijnens, Pallante, Prades '96 & '02;

This contribution will likely become the ultimate limitation of the SM prediction.

The muon g-2: Standard Model vs. Experiment

Adding up all the above contribution we get the following SM predictions for a_μ and comparisons with the measured value:

	$a_\mu^{\rm SM} \times 10^{11}$	$\Delta a_\mu \times 10^{11}$	σ
[1]	116 591 793 (60)	287 (87)	3.3
[2]	116 591 778 (61)	302 (88)	3.4
[3]	116 591 807 (72)	273 (96)	2.8
[4]	116 591 828 (63)	252 (89)	2.8
[5]	116 591 991 (70)	89 (95)	0.9

with $a_\mu^{\rm HHO}$(lbl) = 110 (40) × 10^{-11} .

$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM}$.

[1] Eidelman at ICHEP06 & Davier at TAU06 (update of ref. [5]).
[2] Hagiwara, Martin, Nomura, Teubner, PLB649 (2007) 173.
[3] F. Jegerlehner, PhiPsi 08, Frascati, April 2008.
[4] J.F. de Troconiz and F.J. Yndurain, PRD71 (2005) 073008.
[5] Davier, Eidelman, Hoecker and Zhang, EPJC31 (2003) 503 (τ data).

The th. error is now the same (or even smaller) as the exp. one!

The muon g-2 and the bounds on the Higgs mass

MP, W.J. Marciano & A. Sirlin

arXiv:0804.1142 (PRD, to appear)

How do we explain Δa_μ?

- Δa_μ can be explained in many ways: errors in HHO-LBL, QED, EW, HHO-VP, g-2 EXP, HLO; or New Physics.
- Can Δa_μ be due to hypothetical changes in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$$\begin{aligned} a &= \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3}, \;\; s_u < M_Z^2, \\ b &= \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)}, \end{aligned}$$

and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

($\varepsilon>0$), in the range:

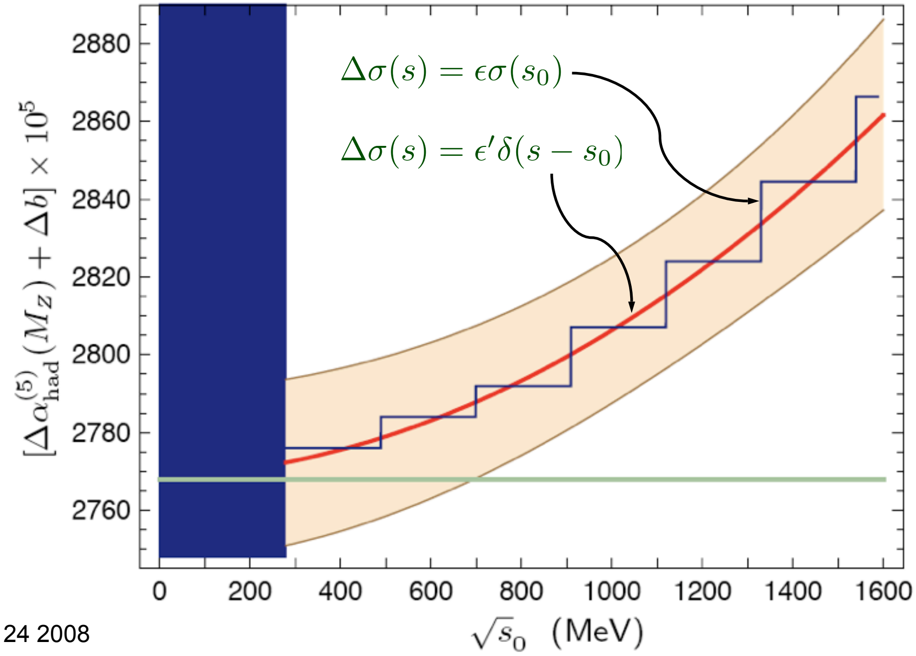
$$\sqrt{(s)} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2]$$

Shifts of a_μ^{HLO} and $\Delta\alpha_{had}^{(5)}(M_Z)$

- If this shift $\Delta\sigma(s)$ in $[\sqrt{s_0}-\delta/2, \sqrt{s_0}+\delta/2]$ is adjusted to bridge the g-2 discrepancy, the value of $\Delta\alpha_{had}^{(5)}(M_Z)$ increases by:

$$\Delta b(\sqrt{s_0},\delta) = \Delta a_\mu \frac{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} g(t^2)\,\sigma(t^2)\,t\,dt}{\int_{\sqrt{s_0}-\delta/2}^{\sqrt{s_0}+\delta/2} f(t^2)\,\sigma(t^2)\,t\,dt}$$

- Adding this shift to $\Delta\alpha_{had}^{(5)}(M_Z) = 0.02768(22)$ [HMNT07] , with $\Delta a_\mu = 302(88) \times 10^{-11}$ [HMNT07], we obtain:



EW Bounds on the SM Higgs mass

- The dependence of SM predictions on the Higgs mass, via loops, provides a powerful tool to set bounds on its value.
- Comparing the theoretical predictions of M_W and $\sin^2\theta_{\rm eff}^{\rm lept}$

  [convenient formulae in terms of M_H, M_{top}, $\Delta\alpha_{had}^{(5)}(M_Z)$ and $\alpha_s(M_Z)$ by Degrassi, Gambino, MP, Sirlin '98; Degrassi, Gambino '00; Ferroglia, Ossola, MP, Sirlin '02; Awramik, Czakon, Freitas, Weiglein '04 & '06]

with

M_W = 80.398 (25) GeV [LEP+Tevatron]

$\sin^2\theta_{\rm eff}^{\rm lept}$ = 0.23153 (16) [LEP+SLC]

and

$\Delta\alpha_{had}^{(5)}(M_Z)$ = 0.02768 (22) [HMNT '07]

M_{top} = 172.6 (1.4) GeV [CDF-D0, Mar '08]

$\alpha_s(M_Z)$ = 0.118 (2) [PDG '06]

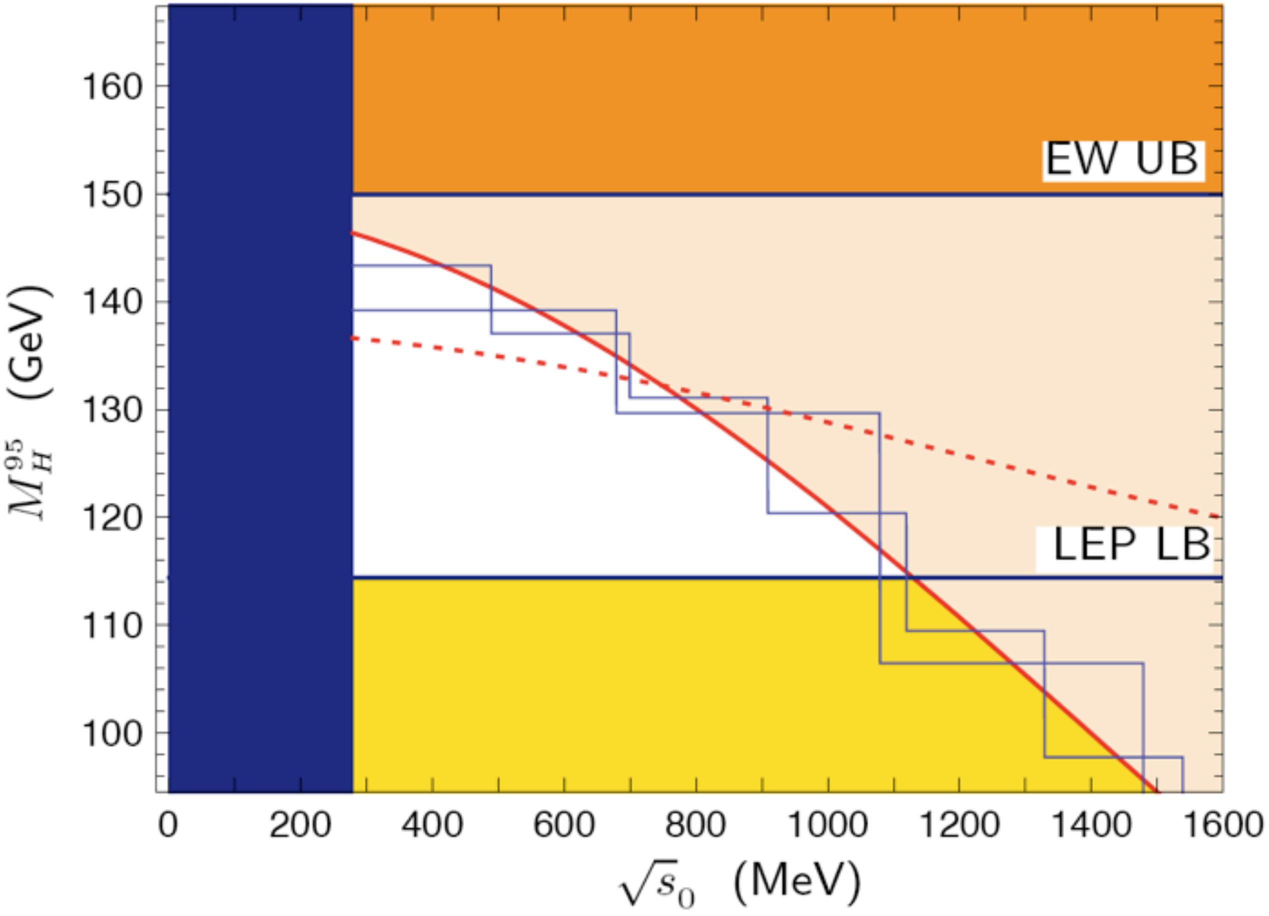
we get

$$M_H = 90\,^{+33}_{-25}\ \text{GeV} \quad \& \quad M_H < 150\ \text{GeV}\ \ 95\%\text{CL}$$

- The value of $\Delta\alpha_{had}^{(5)}(M_Z)$ is a key input of these EW fits...

The muon g-2: connection with the SM Higgs mass

- How much does the M_H upper bound change when we shift $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$ by Δb] to accommodate Δa_μ ?

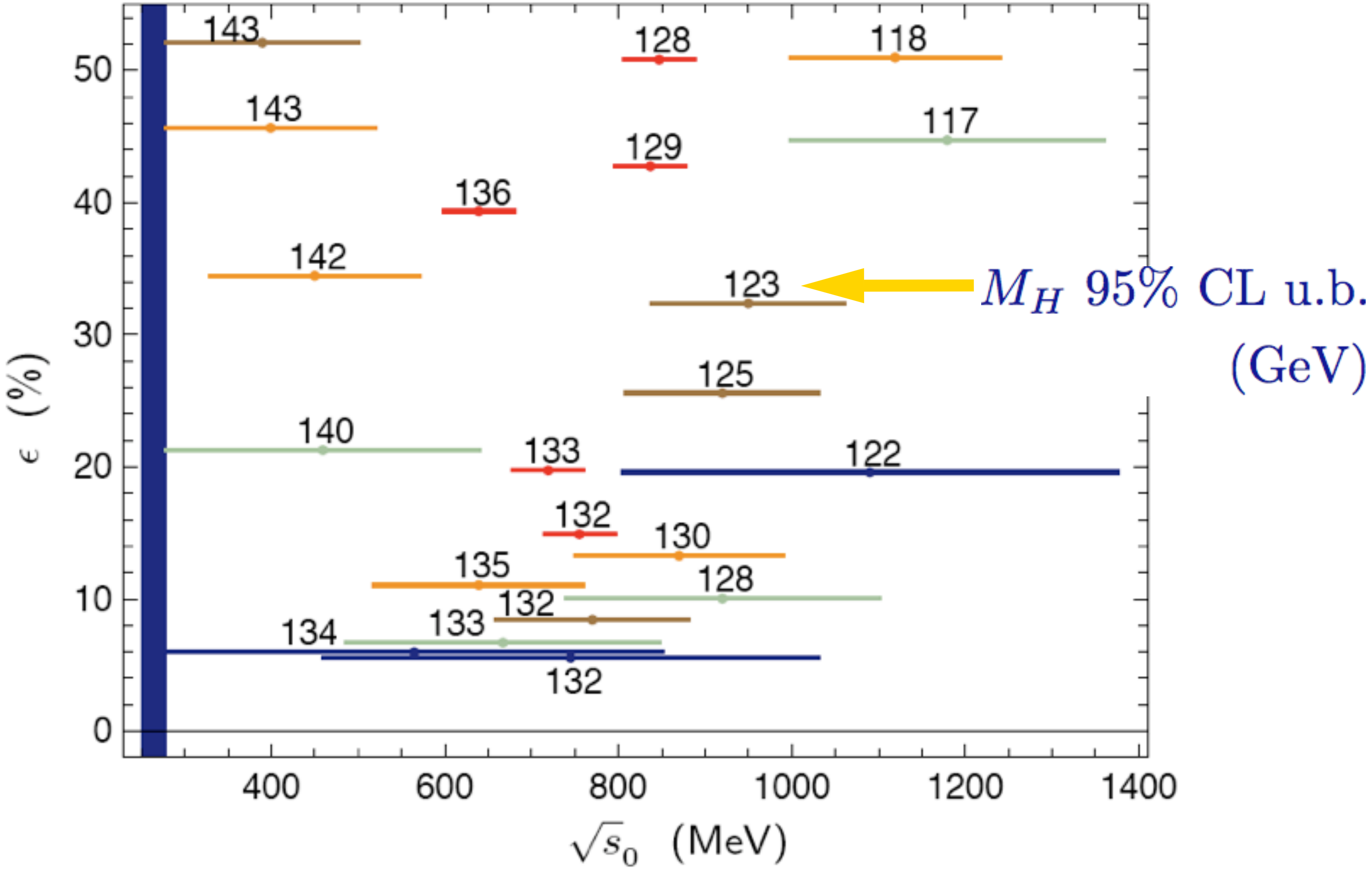


The muon g-2: connection with the SM Higgs mass (2)

- The LEP direct-search lower bound is $M_H^{LB} = 114.4$ GeV (95%CL).
- The hypothetical shifts $\Delta\sigma = \varepsilon\sigma(s)$ that bridge the muon g-2 discrepancy conflict with the LEP lower limit when $\sqrt{s_0} >$ ~1.2GeV (for bin widths δ up to several hundreds of MeV).
- While using tau data in the calculation of a_μ^{HLO} almost solves the muon g-2 discrepancy, it increases the value of $\Delta\alpha_{had}^{(5)}(M_Z)$, leading to $M_H < 138$ GeV (95%CL), in near conflict with M_H^{LB}.
- Recent claim: e^+e^- & tau data consistent below ~1 GeV (after isospin viol. effects & vector meson mixings). We could thus assume that Δa_μ is fixed by hypothetical errors above ~1GeV (where disagreement persists). If so, M_H^{UB} falls below M_H^{LB} !!
- Scenarios where Δa_μ is accommodated without affecting M_H^{UB} are possible, but considerably more unlikely.

How realistic are these shifts $\Delta\sigma(s)$?

- How realistic are these shifts $\Delta\sigma(s)$ when compared with the quoted exp. uncertainties? Study the ratio $\varepsilon=\Delta\sigma(s)/\sigma(s)$:



M_H 95% CL u.b. (GeV)

How realistic are these shifts $\Delta\sigma(s)$? (2)

- The minimum ε is ~ +4%. It occurs if σ is multiplied by $(1+\varepsilon)$ in the whole integration region (!), leading to M_H^{UB} ~ 75 GeV (!!)
- As the quoted exp. uncertainty of $\sigma(s)$ below 1 GeV is ~ a few per cent (or less), the possibility to explain the muon g-2 with these shifts $\Delta\sigma(s)$ appears to be unlikely.
- If, however, we allow variations of $\sigma(s)$ up to ~6% (7%), M_H^{UB} is reduced to less than ~134 GeV (135 GeV). E.g., the ~6% shift in the interval [0.6, 1.2] GeV, required to fix Δa_μ, lowers M_H^{UB} to 130 GeV.
- Reminder: the above M_H upper bounds, like the LEP-EWWG ones, depend on the value of $\sin^2\theta_{\rm eff}^{\rm lept}$. They also depend on M_t & its unc. δM_t. We prepared simple formulae to translate easily M_H upper bounds discussed above into new values corresponding to M_t & δM_t inputs different from those employed here.

Conclusions

- g: Beautiful examples of interplay between theory and experiment: g_e probed at <ppt → α and extraordinary test of QED's validity; g_μ probed at <ppb → test of the full SM and great opportunity to unveil (or just constrain) "New Physics" effects!

- The discrepancy Δa_μ is more than 3 σ if e^+e^- data are used (with tau data, the deviation is only ~ 1 σ). QED and EW terms solid and ready for E969! HLO will continue improving... LBL??

- Δa_μ can be due to New Physics, or to problems in a_μ^{SM} (or a_μ^{EXP}!). Can it be due to errors in the hadronic $\sigma(s)$? An hypothetical increase $\Delta\sigma(s)$ could bridge Δa_μ, leading however to a decrease on the EW upper bound on the SM Higgs mass M_H ...

- By means of a detailed analysis we conclude that solving Δa_μ via an increase of $\sigma(s)$ is unlikely in view of current exp. error estimates. However, if this turns out to be the solution, then the M_H upper bound drops to about 130 GeV which, in conjunction with the LEP 114 GeV direct lower limit, leaves a rather narrow window for M_H.

The End