A Singularity Problem with F(R) Dark Energy

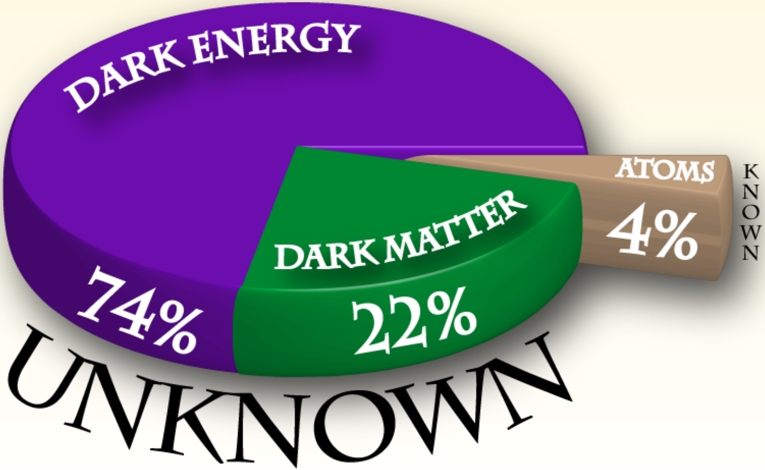
arXiv:0803.2500

Andrei Frolov

Department of Physics
Simon Fraser University

Institute for the Physics and
Mathematics of the Universe
University of Tokyo, Kashiwa, Japan
28 August 2008

What's the Matter with Cosmology?



Maybe it's Gravity we Don't Understand...

What if instead of curvature in Einstein-Hilbert action we had

$$S = \int \left\{ \frac{f(R)}{16\pi G} + \mathscr{L}_{\rm m} \right\} \sqrt{-g}\, d^4x$$

UV modification:

$$f(R) = R + \frac{R^2}{M^2}$$

Starobinsky (1980)

IR modification:

$$f(R) = R - \frac{\mu^4}{R}$$

Capozziello et. al. [astro-ph/0303041]
Carroll et. al. [astro-ph/0306438]

For F(R) theory to make sense we need:

- $f' > 0$ – otherwise gravity is a ghost
- $f'' > 0$ – otherwise gravity is a tachyon

After a Rocky Start... A New Trouble?

Linearly stable models have been found:

Hu and Sawicki [0705.1158]

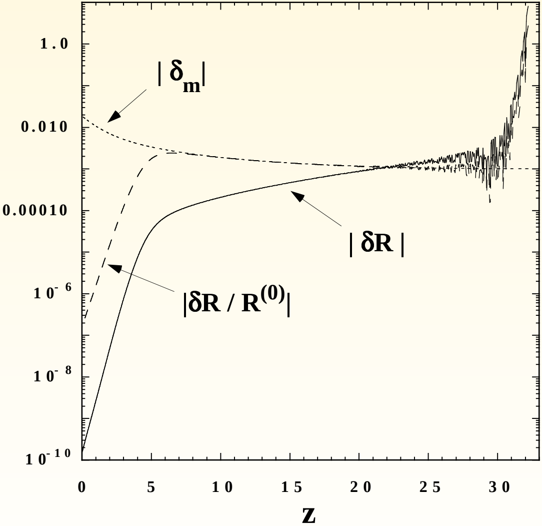
Starobinsky [0706.2041]

various "designer" models

LSS growth studied:

Pogosian & Silvestri [0709.0296]

But something is amiss...



Tsujikawa [0709.1391]

Why Doesn't F(R) Dark Energy Work?

IR <- 60 orders of magnitude -> UV

IR modification

$$f(R) = R - \frac{\mu^4}{R}$$

$$\phi = \frac{\mu^4}{R^2}$$

damage to UV

IR UV

O(1)

Field Equations in F(R) Gravity

- Vary the action with respect to the metric:

$$S = \int \left\{ \frac{f(R)}{16\pi G} + \mathscr{L}_{\mathrm{m}} \right\} \sqrt{-g}\, d^4x$$

- Einstein equations turn into a fourth-order equation:

$$f' R_{\mu\nu} - f'_{;\mu\nu} + \left(\Box f' - \frac{1}{2} f \right) g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$$

- A new scalar degree of freedom $\phi \equiv f' - 1$ appears:

$$\Box f' = \frac{1}{3}(2f - f'R) + \frac{8\pi G}{3} T$$

- Can rewrite fourth-order field equation as two second order ones!

A New Scalar Degree of Freedom

- Equation for $\phi \equiv f' - 1$ is just a scalar wave equation:

$$\Box\phi = V'(\phi) - \mathscr{F}$$

- Matter directly drives the field ϕ by a force term:

$$\mathscr{F} = \frac{8\pi G}{3}(\rho - 3p)$$

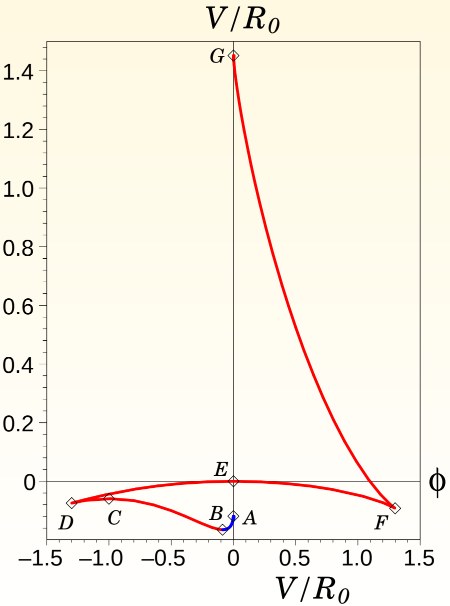
- Effective potential can be found by integrating

$$V'(\phi) \equiv \frac{dV}{d\phi} = \frac{1}{3}(2f - f'R)$$

- In practice, easier to obtain in parametric form:

$$\frac{dV}{dR} \equiv \frac{dV}{d\phi}\frac{d\phi}{dR} = \frac{1}{3}(2f - f'R)f''$$

Disappearing Cosmological Constant Model



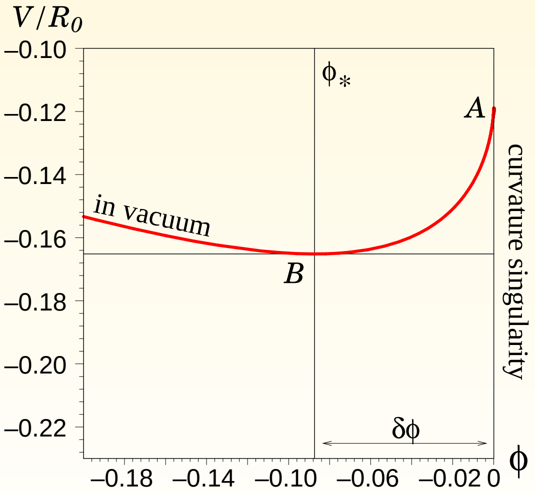
Starobinsky [0706.2041]

$$f(R) = R + \lambda\left[\left(1+R^2\right)^{-1} - 1\right]$$

$$\phi = -\frac{2\lambda R}{(1+R^2)^2}$$

A	singularity	$(R = +\infty)$
B	stable dS min	$(\ f' = 0\ )$
C	unstable dS max	$(\ f' = 0\ )$
D	critical point	$(\ f'' = 0\ )$
E	flat spacetime	$(\ f' = 0\ )$
F	critical point	$(\ f'' = 0\ )$
G	singularity	$(R = -\infty)$

SINGULARITY IS FINITE DISTANCE AWAY!



$$U(\phi) = V(\phi) + \mathscr{F}(\phi_* - \phi)$$

in large R limit:

$$f(R) = R + \Lambda + \frac{1}{R^\alpha} \sum_{n=0}^{\infty} \frac{\mu_n}{R^n}$$

$$\phi \equiv f' - 1 \simeq -\frac{\alpha \mu_0}{R^{\alpha+1}}$$

$$\frac{dV}{dR} \simeq \frac{R f''}{3} = \frac{\alpha(\alpha+1)\mu_0}{3\, R^{\alpha+1}}$$

weak power-law singularity:

$$V(\phi) \simeq \text{const} - \frac{(\alpha+1)\mu_0}{3\,|\alpha\mu_0|^\gamma}\, |\phi|^\gamma$$

$$\gamma = \frac{\alpha}{\alpha+1}$$

Cosmology in F(R) Gravity

- Homogeneous flat cosmology is described by FRW metric:

$$ds^2 = -dt^2 + a^2(t)\,d\vec{x}^2$$

- Scalar degree of freedom looks as usual (albeit with a force term):

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \mathscr{F}$$

- What about Friedman equation? It looks strange...

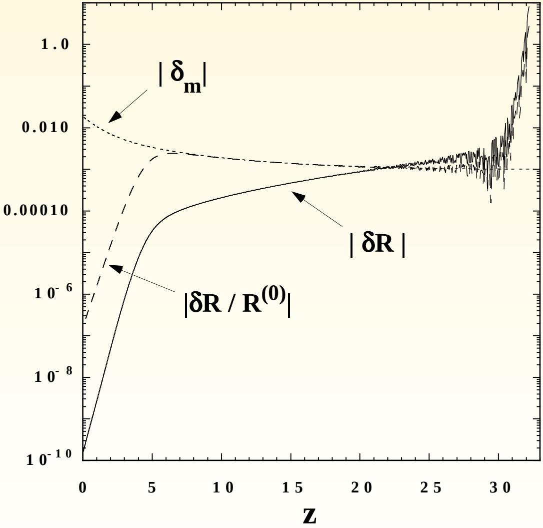
$$3H(f')^{\cdot} - 3\frac{\ddot{a}}{a}f' + \frac{1}{2}f = 8\pi G\rho$$

- ... but it isn't! Eliminating $\ddot{a}$ in favor of $R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)$ does the trick:

$$H^2 + (\ln f')^{\cdot} H + \frac{1}{6}\frac{f - f'R}{f'} = \frac{8\pi G}{3f'}\rho$$

Mechanical Analogy: a Ball in a Bowl

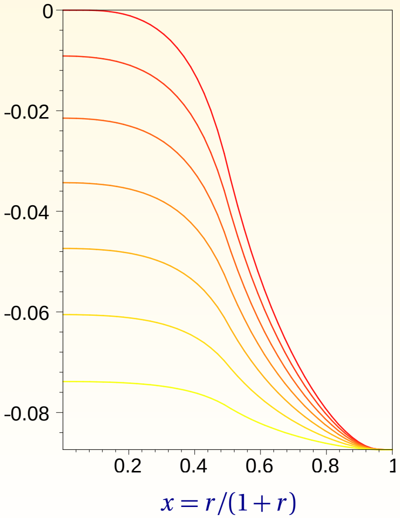
Now we Understand What's Going On Here!



Tsujikawa [0709.1391]

Bigger Problem: Singular Compact Objects!

scalar DOF ϕ



Potential well of a compact object:

$$\Delta\phi = -\frac{8\pi}{3}G\rho + \underbrace{V'(\phi)}_{\text{negligible}}$$

$$\Delta\Phi = 4\pi G\rho$$

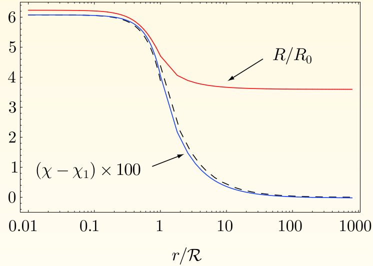
Excitations of f(R) degree of freedom ϕ and Newtonian potential Φ are related:

$$\phi \approx \phi_* - \frac{2}{3}\Phi$$

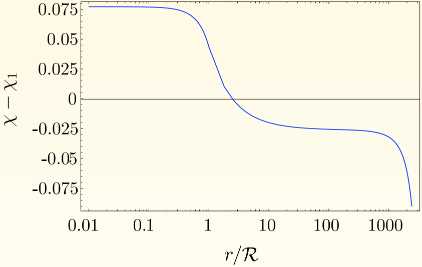
Reach singularity if $\delta\phi \lesssim \frac{1}{3}$!

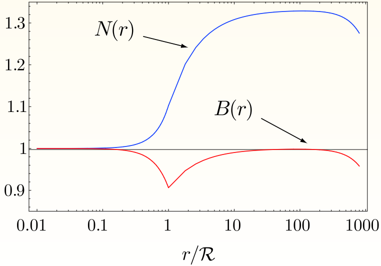
Kobayashi & Maeda (0807.2503)

shallow potential well



deep potential well

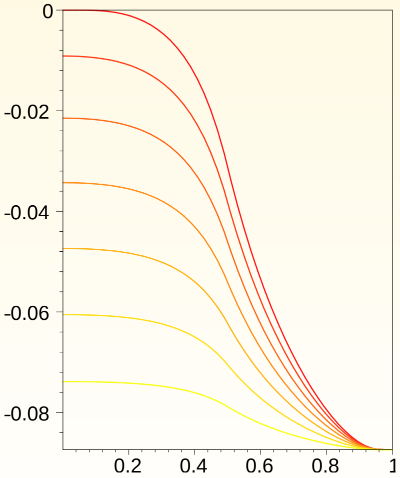




no regular static solution!

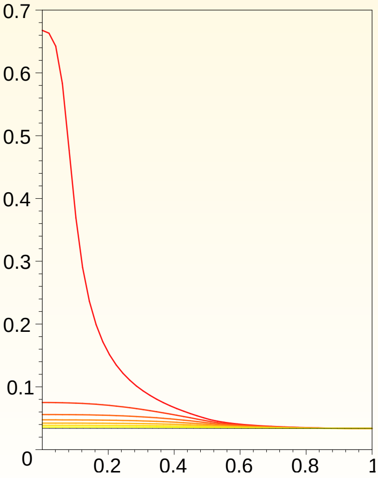
How do I Lose the Regular Solution Branch?

scalar DOF ϕ



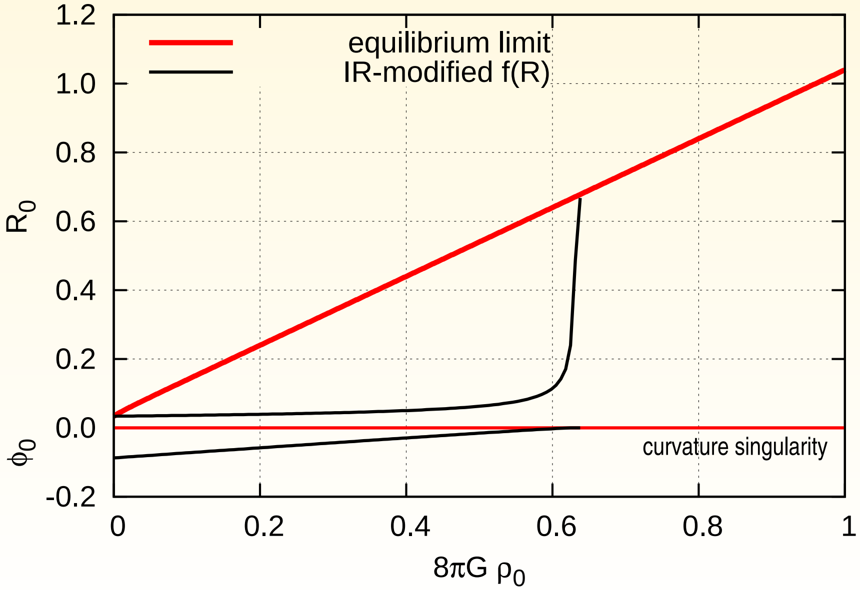
mildly non-linear

curvature R

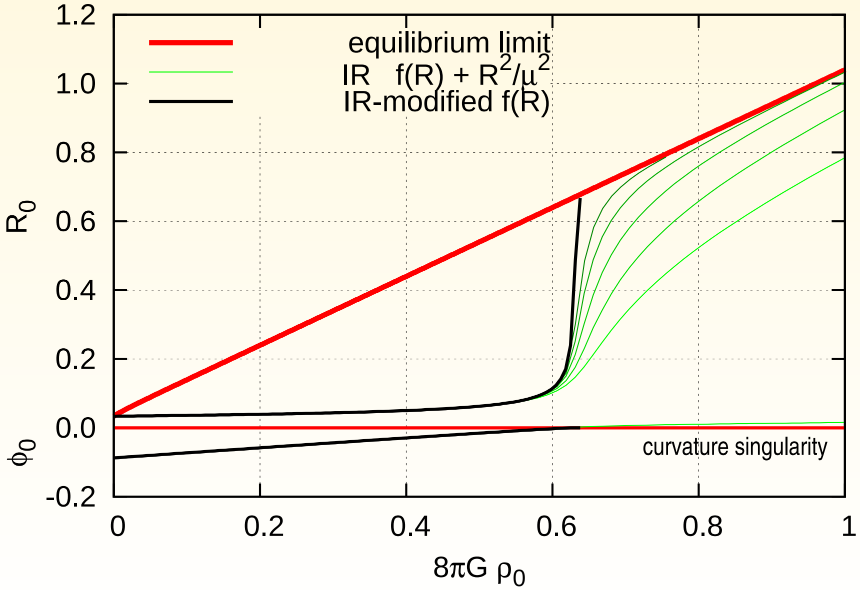


very non-linear

How do I Lose the Regular Solution Branch?



Can UV Completion Save the Day?



Not quite yet!

But we are forced to confront UV-completion, and even if we fix it we might not get Einstein gravity...

Need to understand how bad it really is!