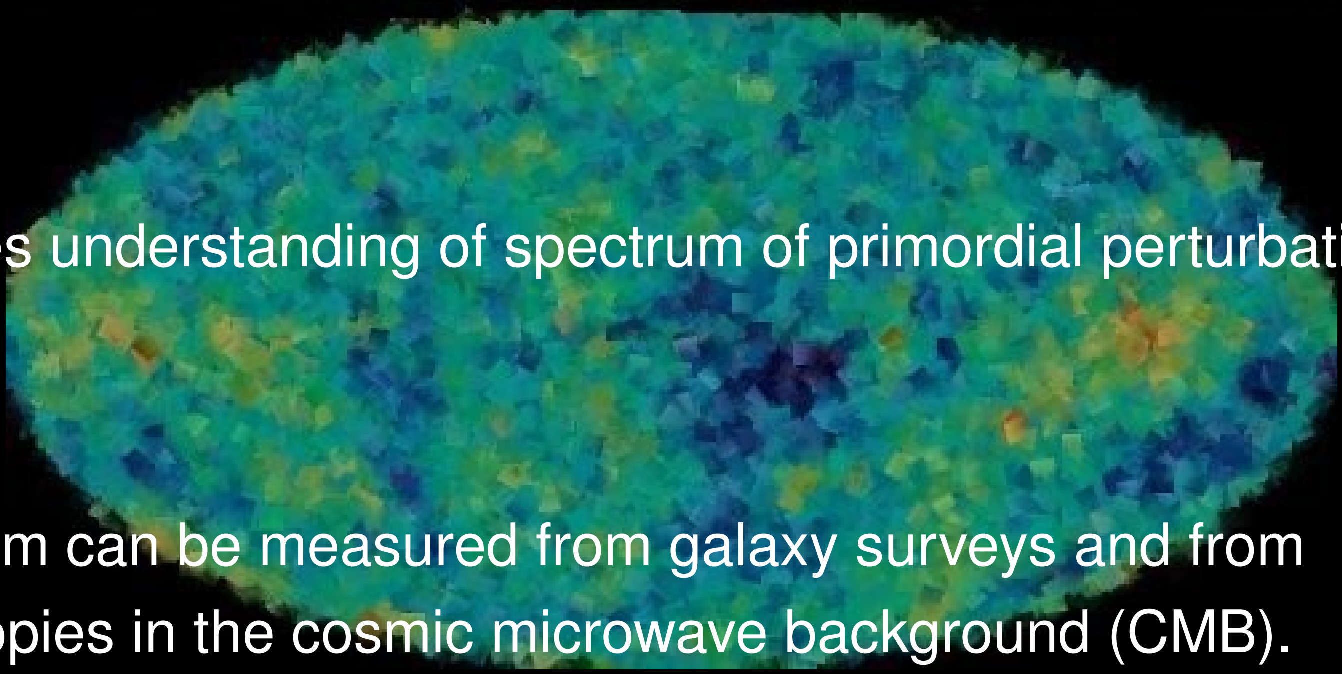
What We Know
(and may ever know)
About Inflation

Brian Powell
IPMU

September 18, 2008

Introduction

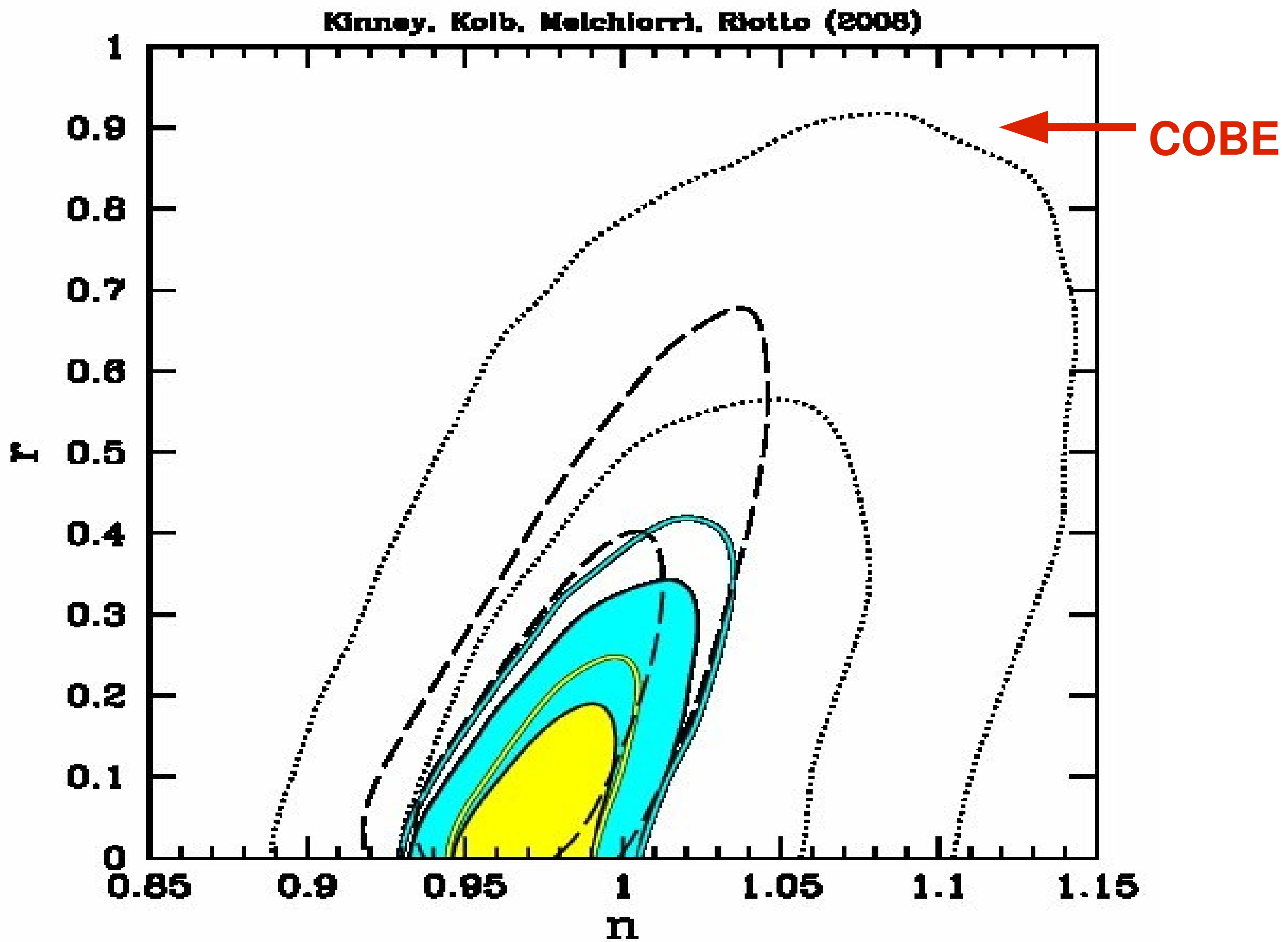


- We would like to know what drove inflation. What is $\mathcal{L}$?

- Requires understanding of spectrum of primordial perturbations, $P(k)$

- Spectrum can be measured from galaxy surveys and from anisotropies in the cosmic microwave background (CMB).

Progress


Kinney, Kolb, Melchiorri, Riotto (2008)
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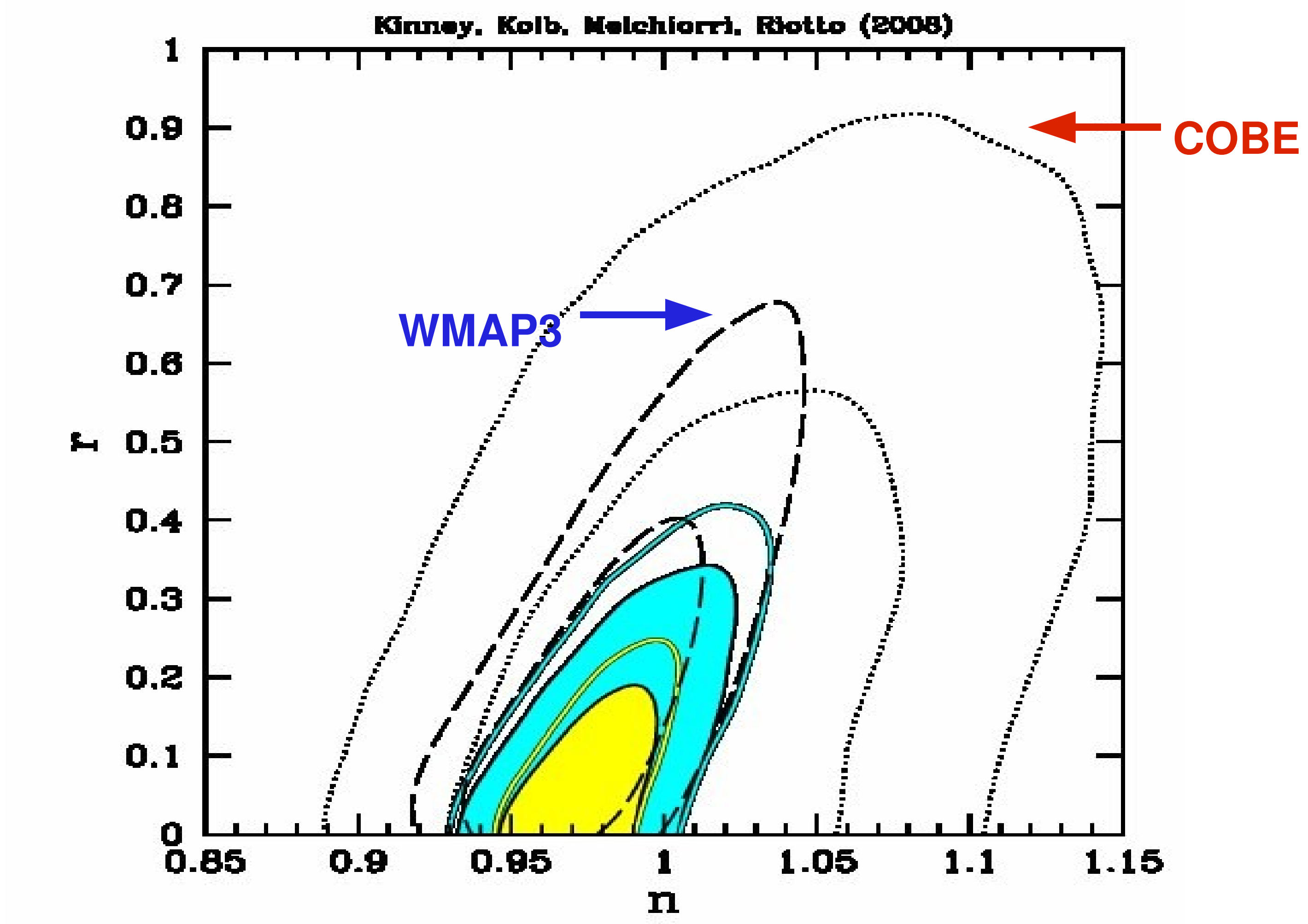
COBE

Progress



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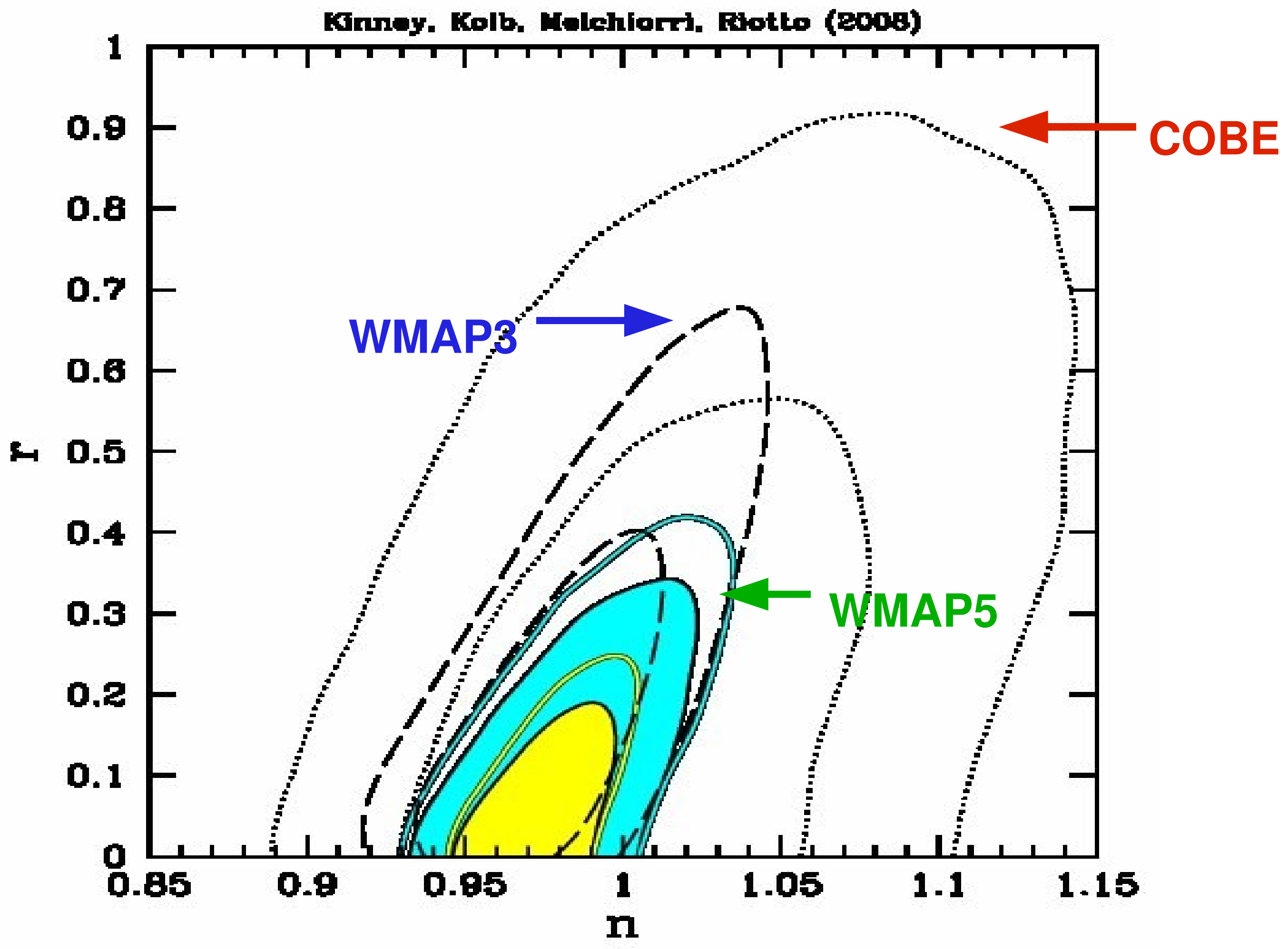

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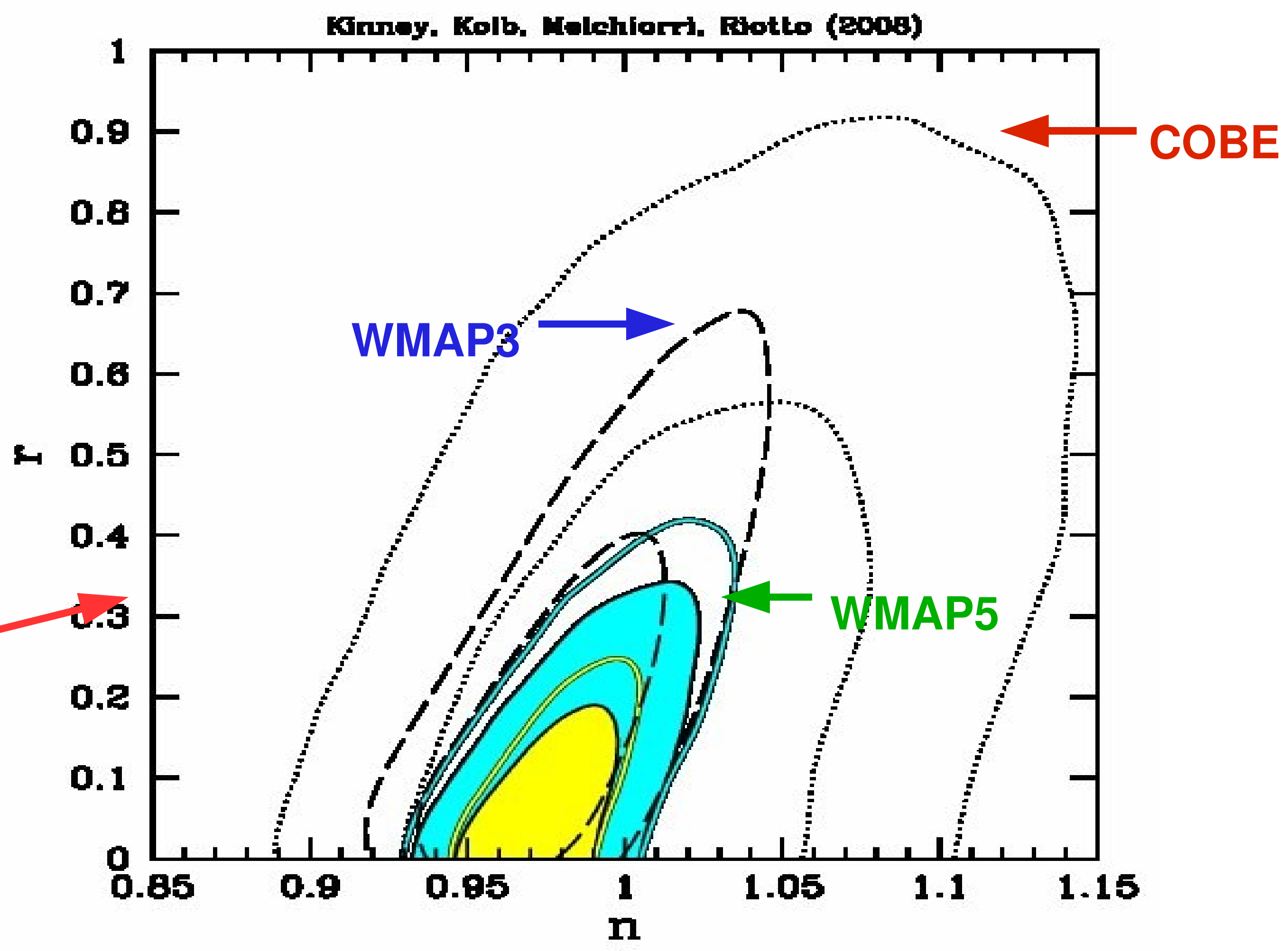
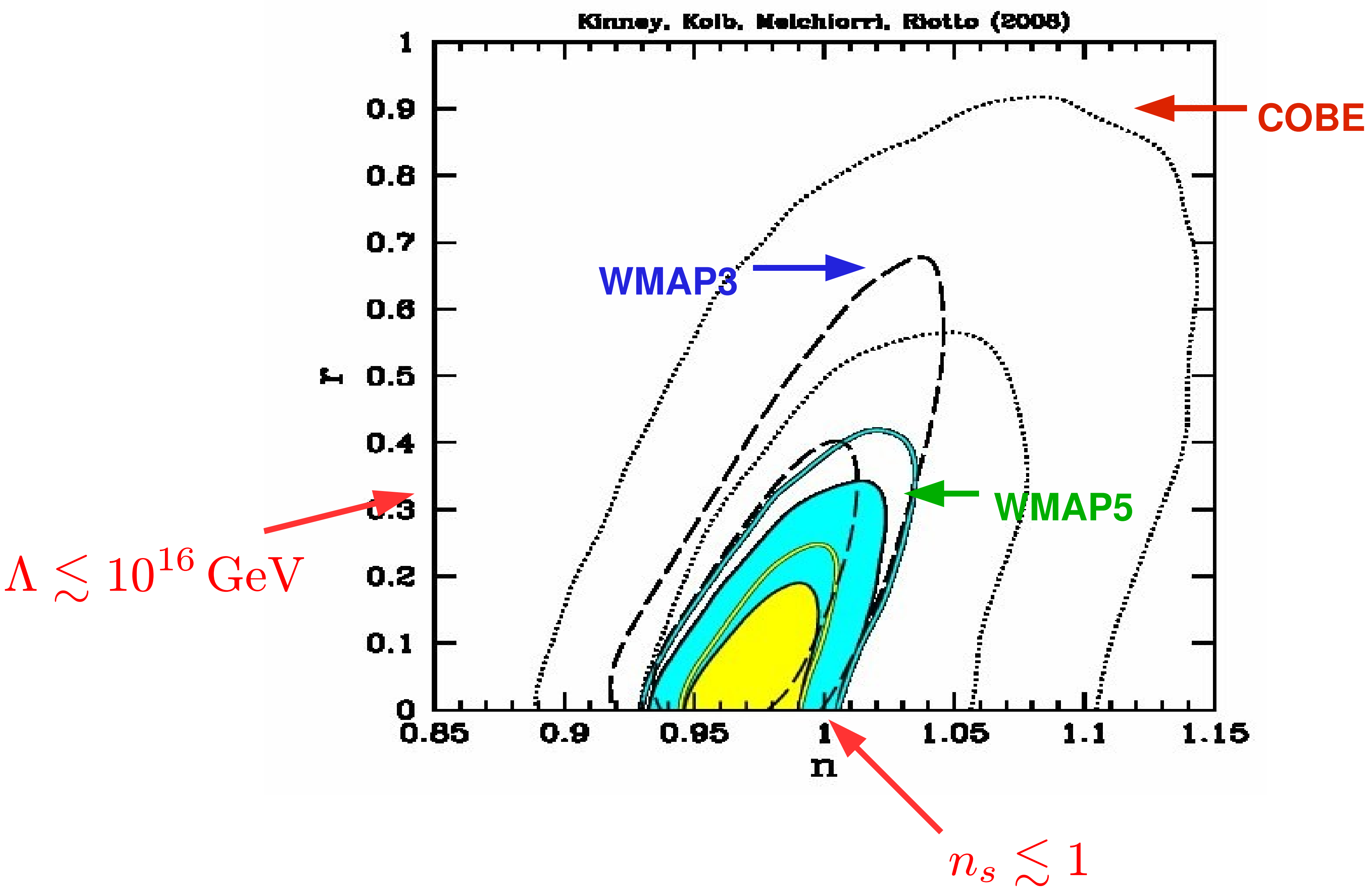


$\Lambda \lesssim 10^{16}$ GeV

Progress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- These results are consistent with the simplest models of *single field, slow roll inflation*.
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What else might the data be consistent with??

Outline

- Background:
  - Inflation
  - Primordial Perturbations
- Methodology
  - Flow Formalism
  - Evolution of Fluctuations
  - Results
- A possibility for the future...

The Expanding Universe

- The inflationary universe is homogeneous and isotropic.

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2$$

- Equations of motion:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3m_{\mathrm{Pl}}^2}\rho \qquad \frac{\ddot{a}}{a} = -\frac{4\pi}{3m_{\mathrm{Pl}}^2}(\rho + 3p)$$

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Matter: $\rho \propto a^{-3}(t)$

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Any fluid with $p/\rho = w < -1/3$

Inflation from Scalar Fields

- Considered a homogeneous, minimally coupled scalar field:

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi)$$

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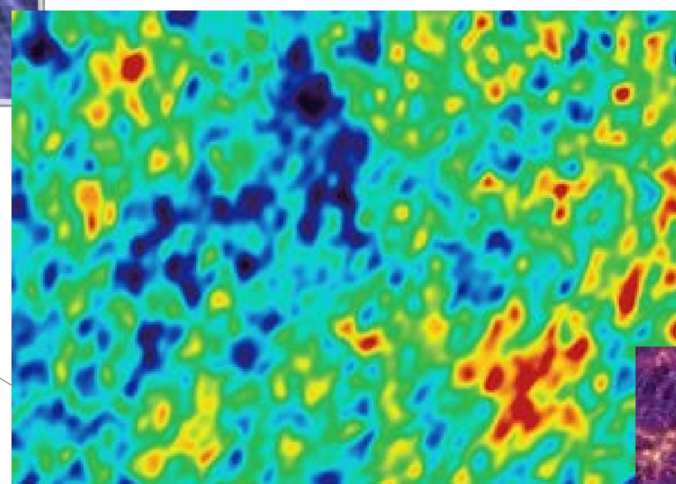
Slow Roll

Primordial Perturbations

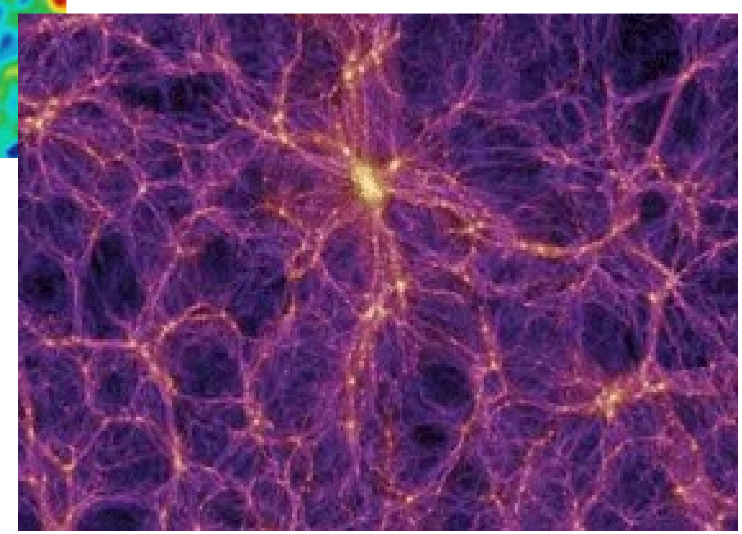
quantum fluctuations

temperature anisotropies



Inflation

large-scale structure



Quantum Fluctuations

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inflaton → density perturbations (scalar)

gravity → gravitational waves (tensorial)

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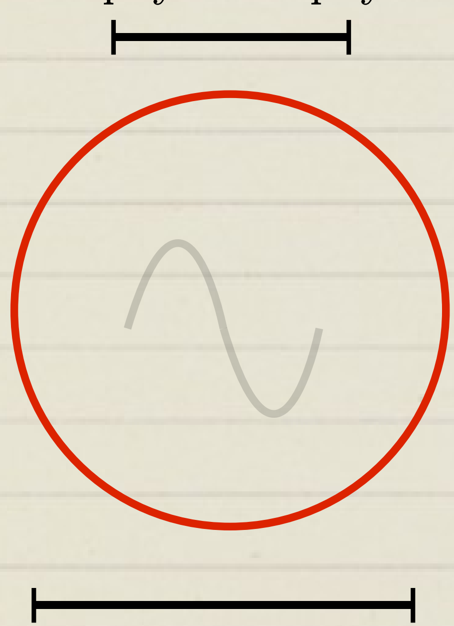

Fourier mode, $\delta\phi_k$

$k_{\rm phys} > H$

d_H

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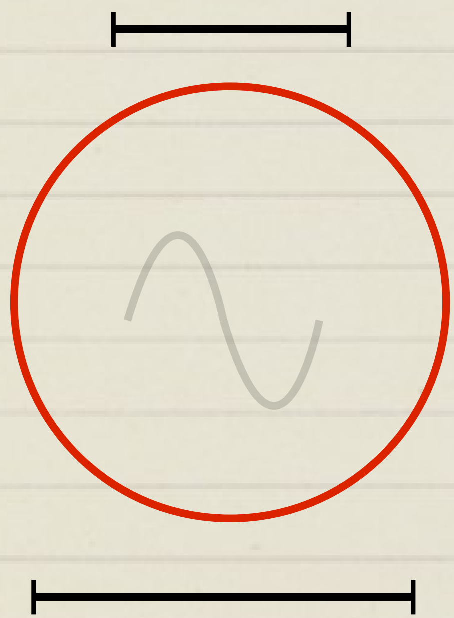

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$$\frac{d_H}{\ell_{\rm phys}} = \frac{1}{aH} = \frac{1}{\dot{a}}$$

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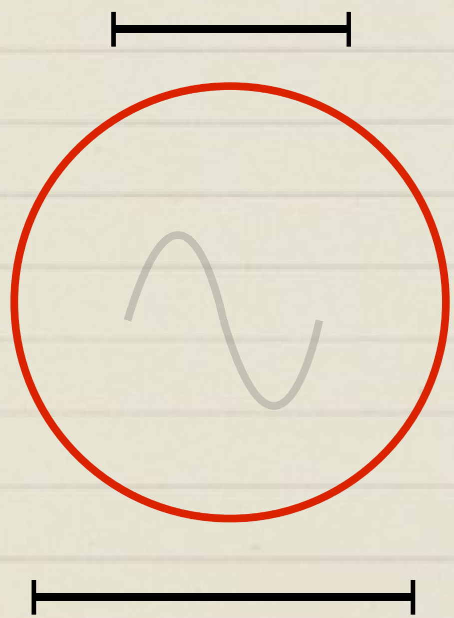

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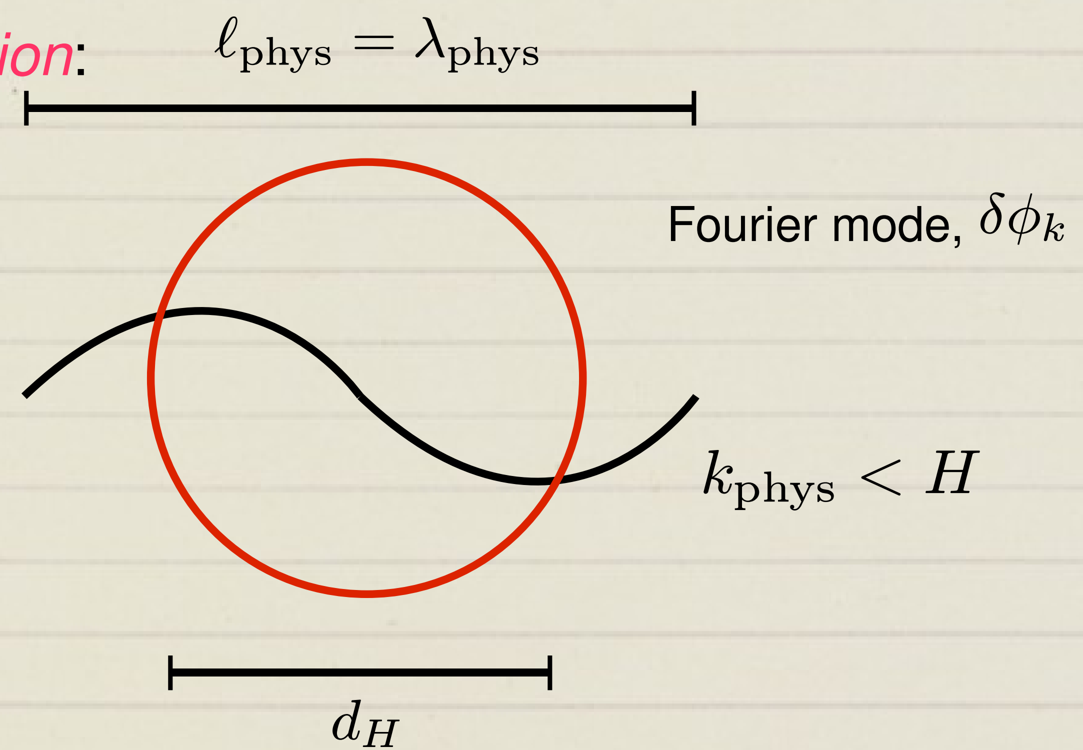
- *Classical Perturbation*:



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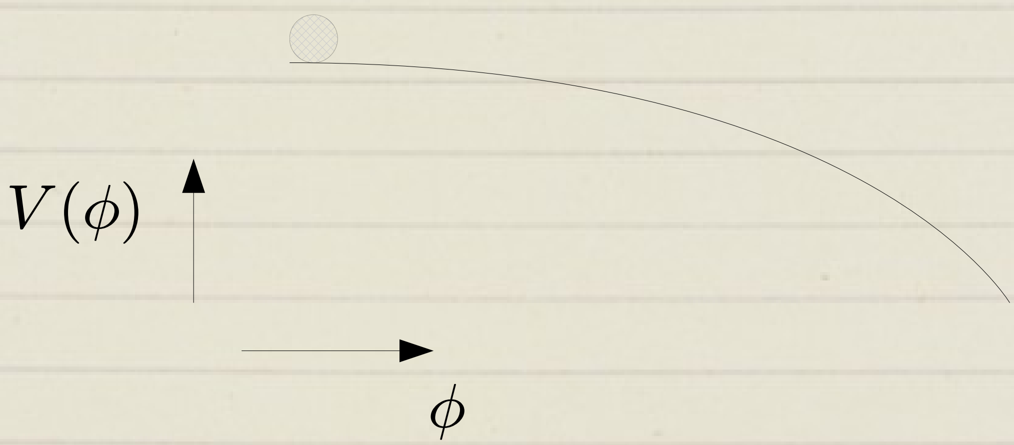

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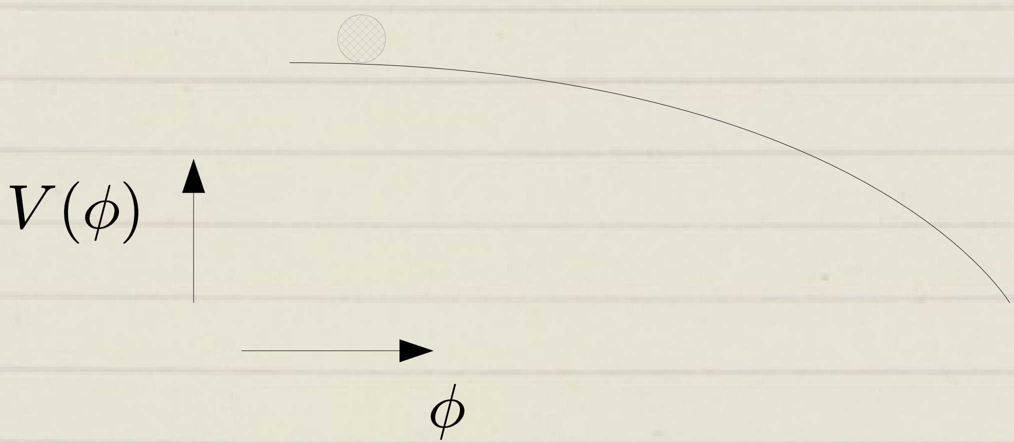

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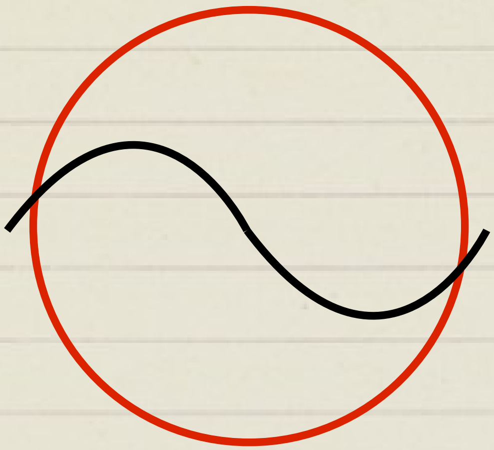

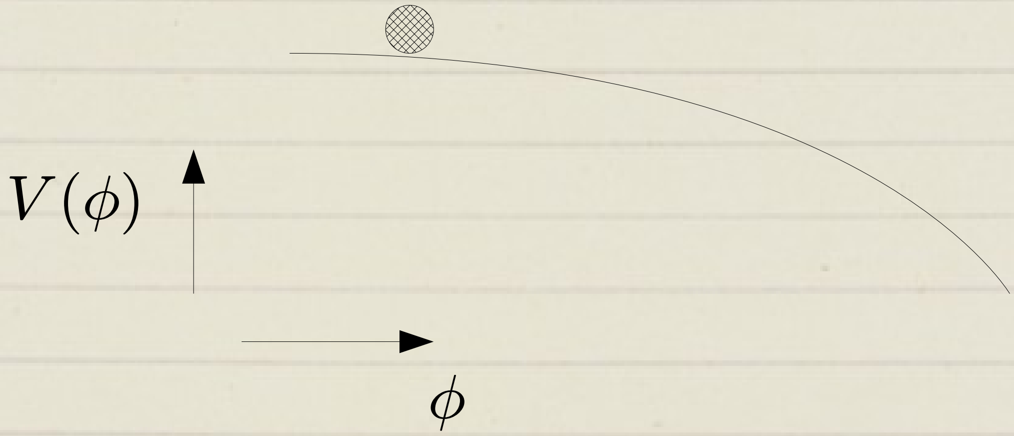

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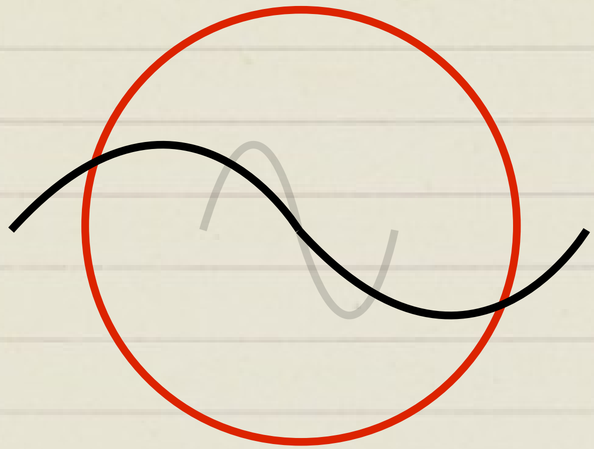

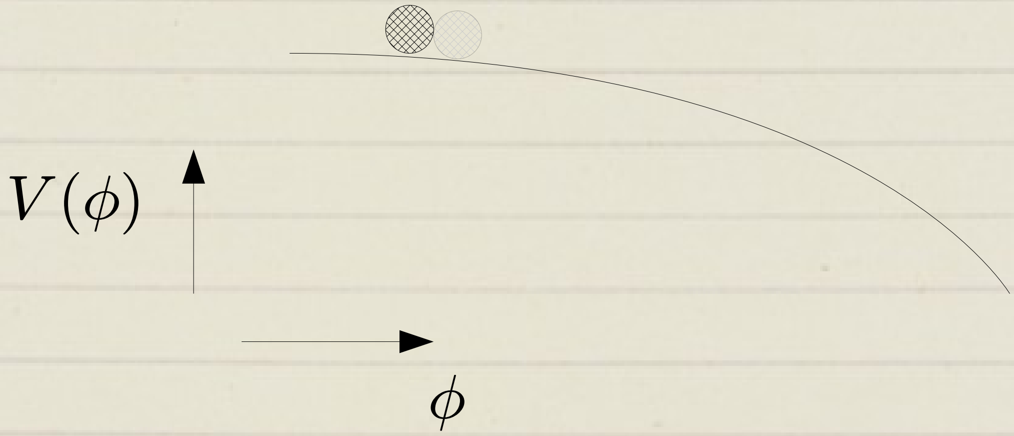

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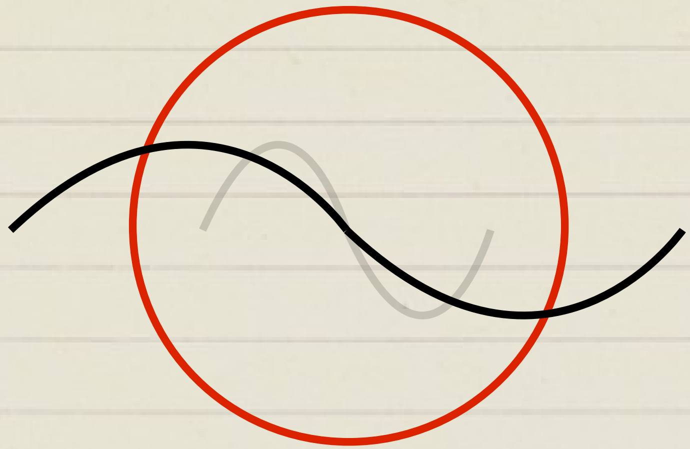

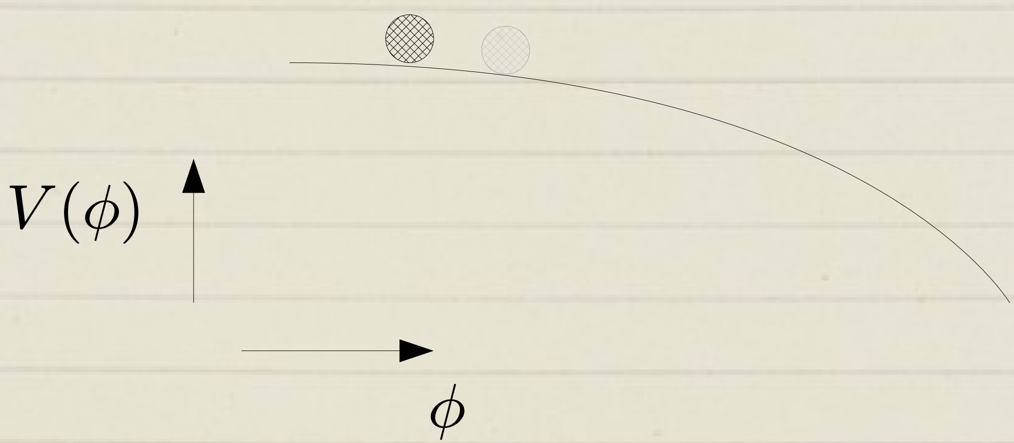

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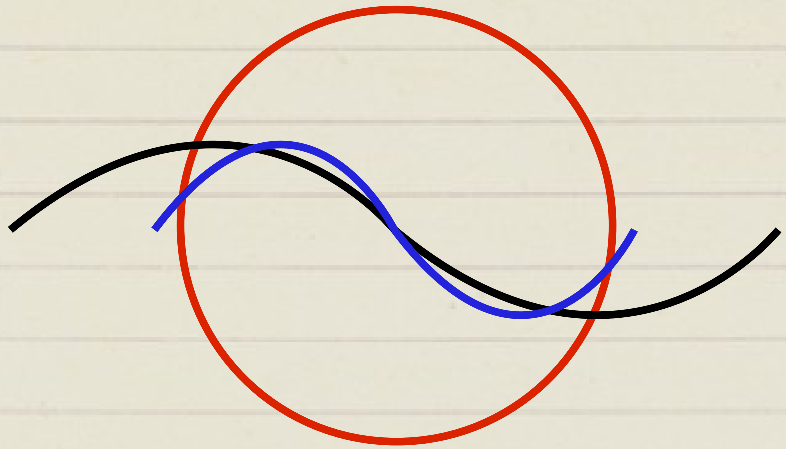

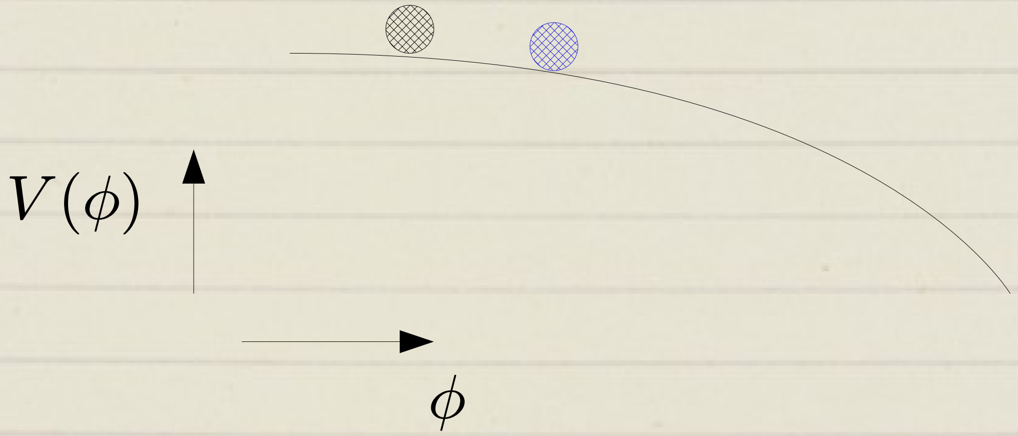

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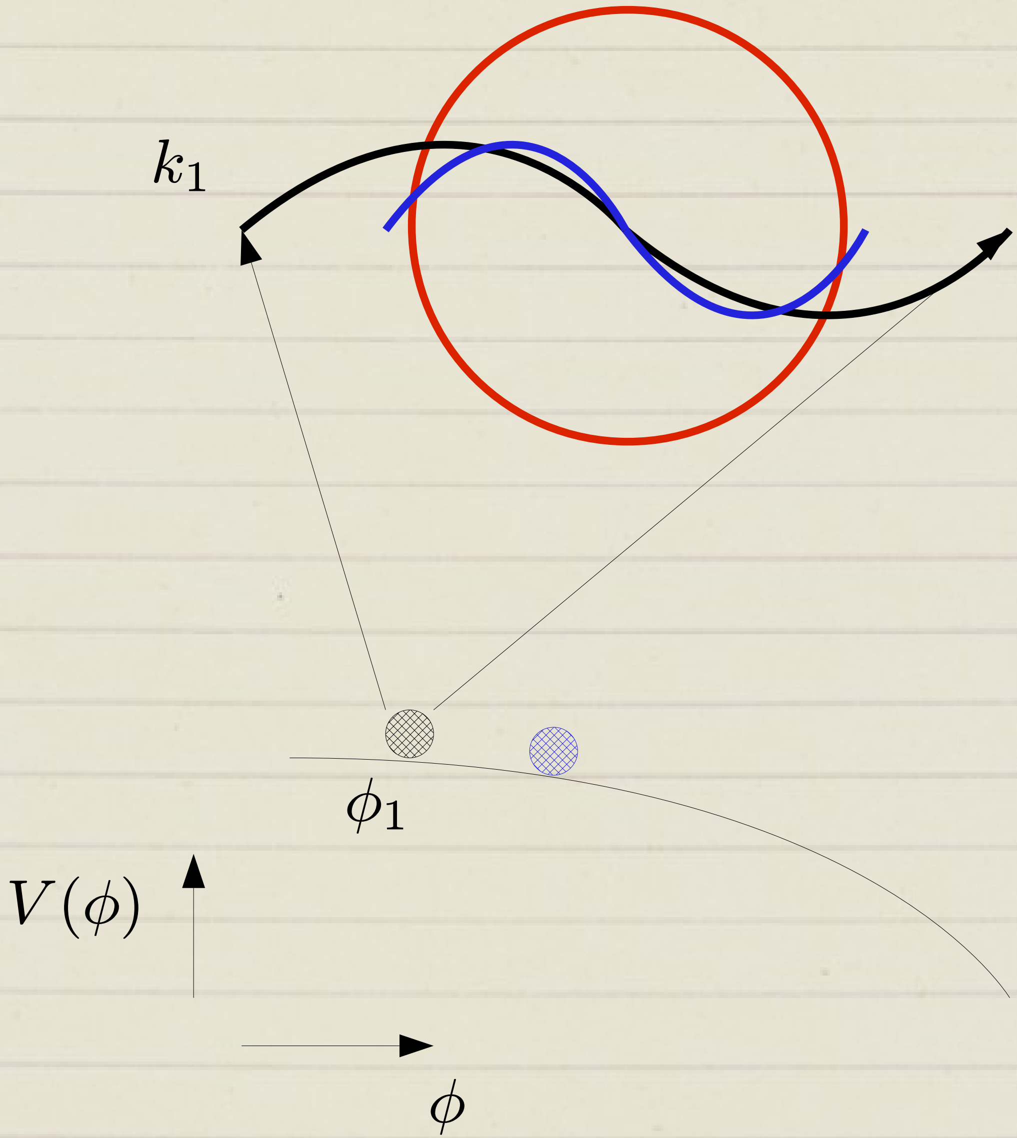

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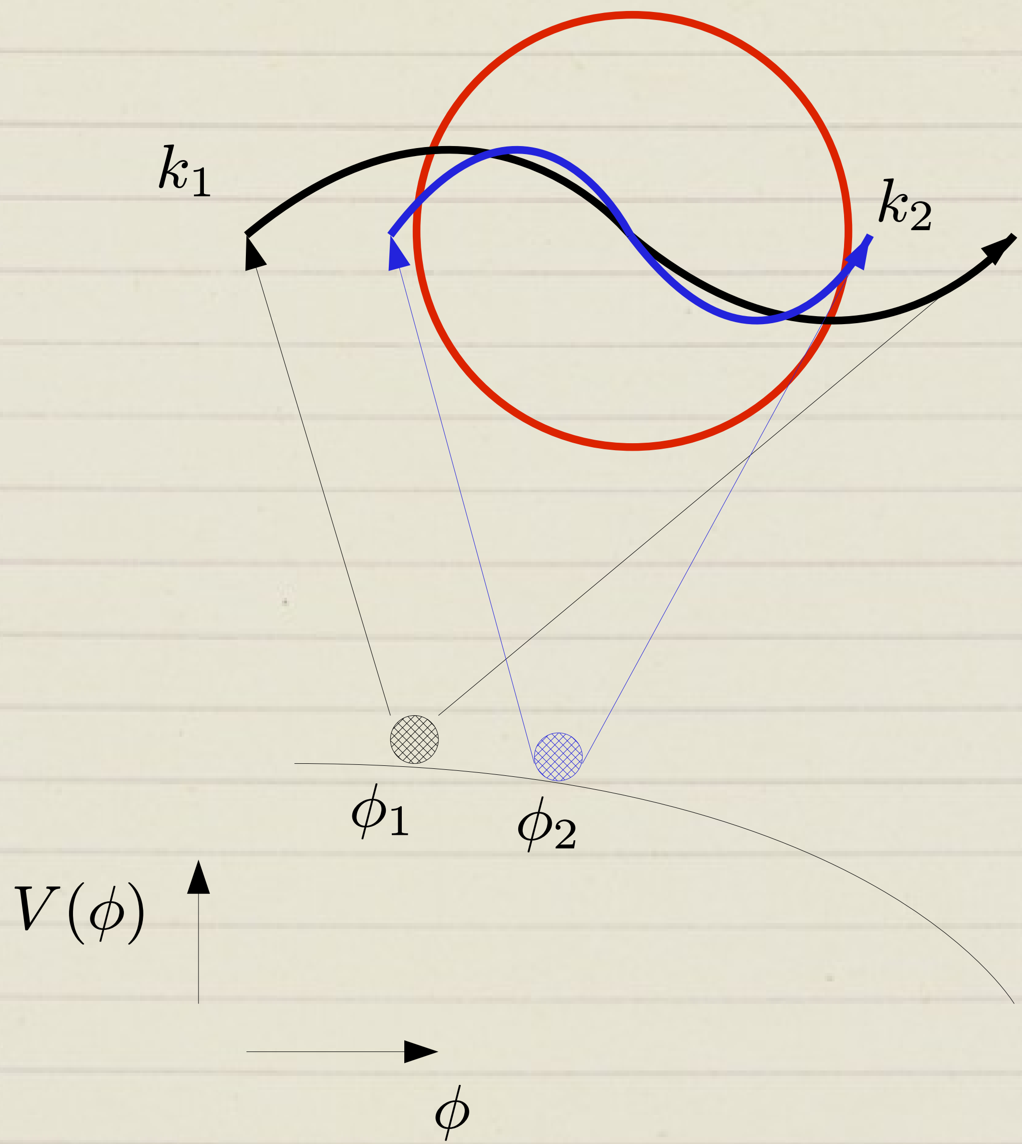

Power Spectrum

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- Amplitude of scalar (curvature) perturbations:

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Power Spectrum

- Typical parameterization of tensor spectrum, $P_T(k) = A_T k^{n_T}$
- Amplitude of tensors:

$$P_T(k) = 16 \frac{H^2}{\pi} \bigg|_{k=aH}$$

- Define tensor/scalar ratio: $r \equiv \frac{P_T}{P_{\mathcal{R}}} = 16\epsilon$

What We Learn From the Spectrum

- Observables:

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During slow roll:

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$$P_{\mathcal{R}}(k_0),\ n_s(k_0),\ r(k_0)$$

$$\Updownarrow$$

$$V(\phi_0),\ V'(\phi_0),\ V''(\phi_0)$$

Recap

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- However, we seek to broaden the scope by investigating more general inflation models.
- How does one systematically test “more general” inflation models?
- How does one determine the resulting power spectra?

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power law inflation

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$$\frac{d({}^\ell\lambda_H)}{dN} = \left[(\ell-1)\eta - \ell\epsilon\right]{}^\ell\lambda_H + {}^{\ell+1}\lambda_H$$

Flow Formalism

- The flow formalism is a model-independent representation of inflationary dynamics.
- It employs Monte Carlo to generate large numbers of different inflationary trajectories, $H(\phi)$.

$$\epsilon = \frac{1}{H}\frac{dH}{dN}$$

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Flow Equations

Flow Formalism

- In practice, the flow system is truncated at some order M, so that

$$^{\ell+1}\lambda_H = 0 \qquad \forall\, \ell \geq M$$

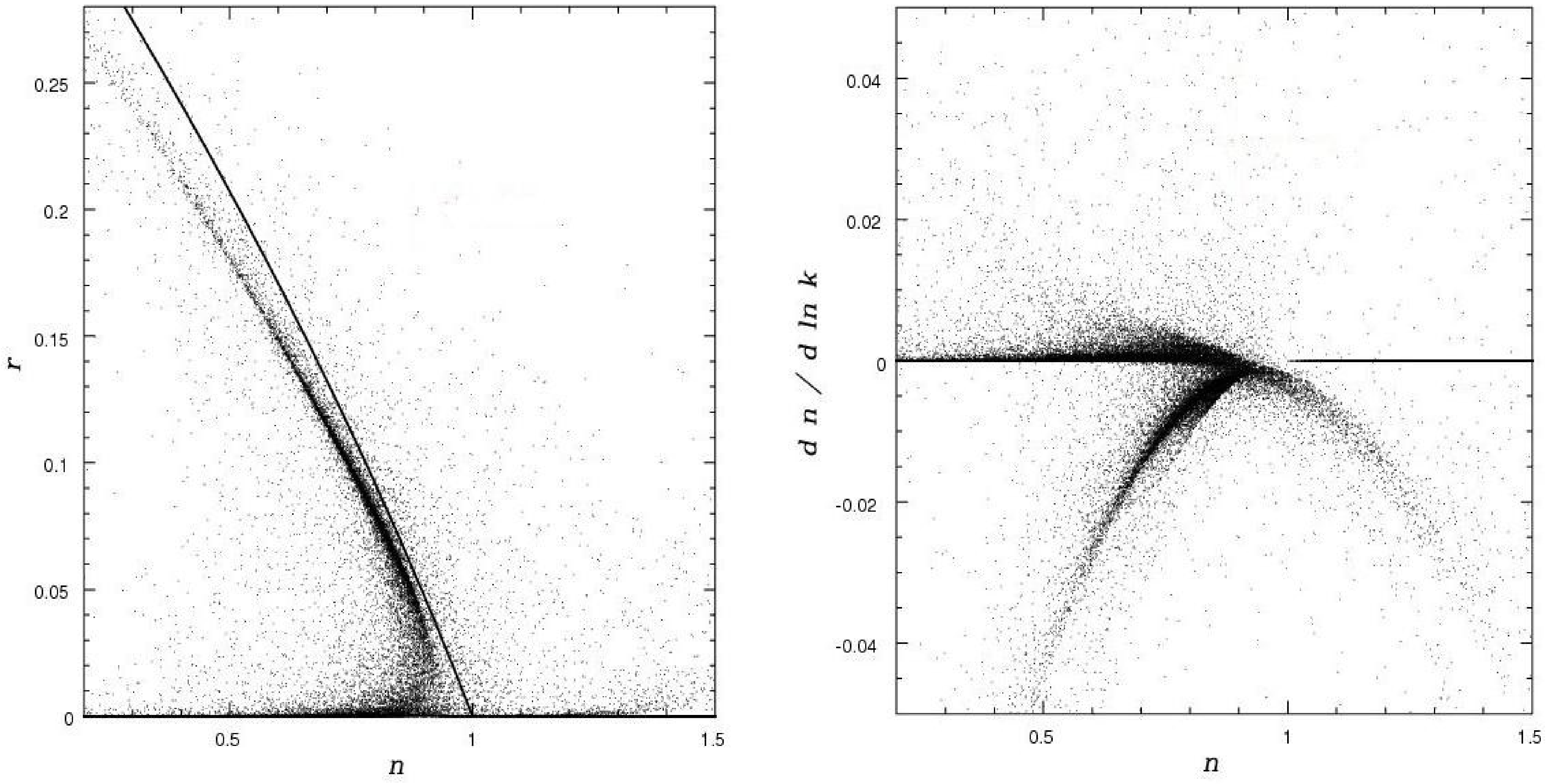
- The resulting function, $H(N)$, represents an *exact* solution of the inflationary equations of motion.

- Solutions are polynomials:

$$H(\phi) = A_1\phi + A_2\phi^2 + \cdots + A_M\phi^M$$

- Keep only those models for which $\Delta N \in [46, 60]$.

Flow Formalism



Flow Formalism

- Given any solution, the potential can be recovered via the Hamilton-Jacobi equation:

$$H(\phi)^2 \left[1 - \frac{1}{3}\epsilon(\phi)\right] = \frac{8\pi}{3m_{\rm Pl}^2} V(\phi)$$

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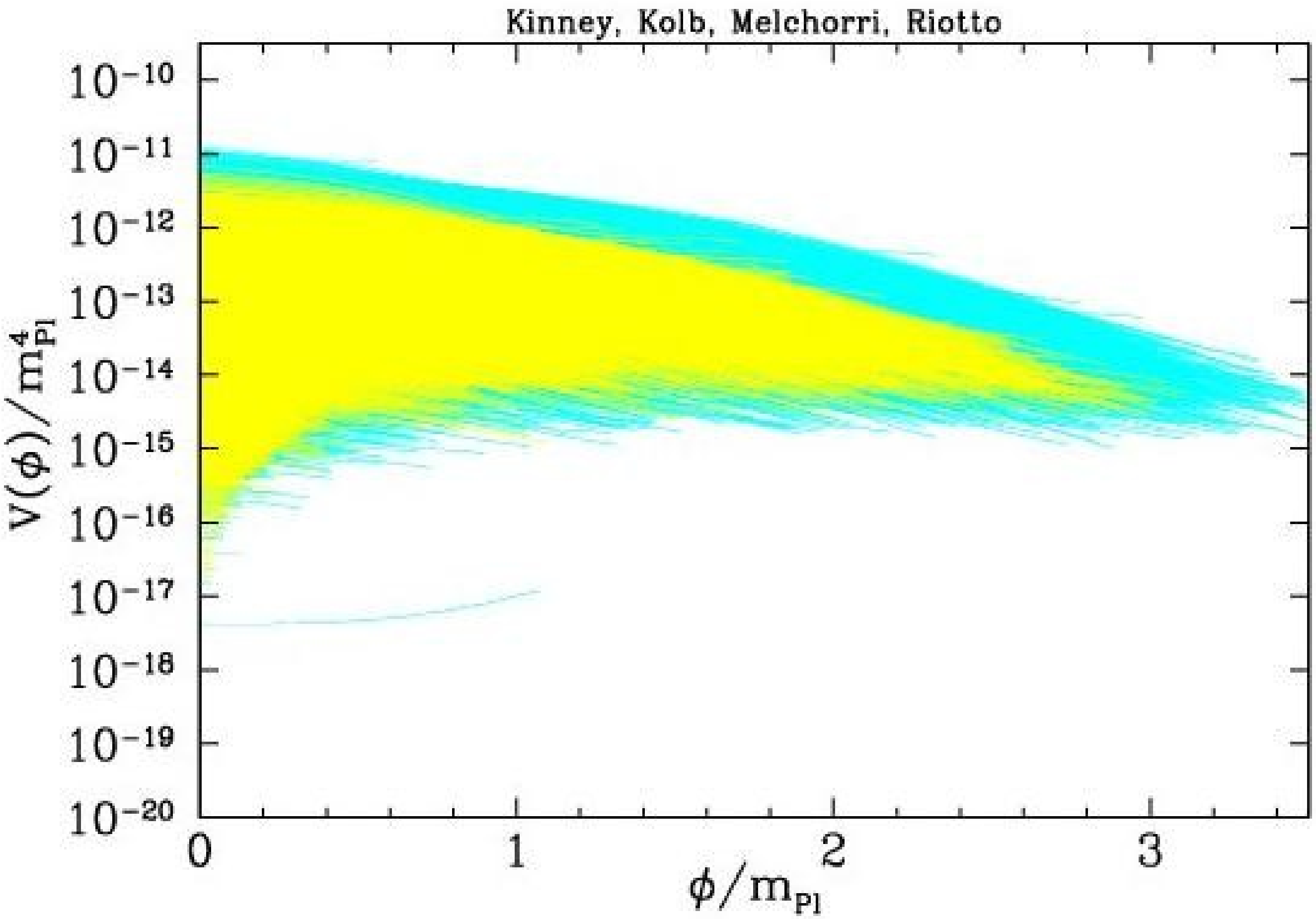

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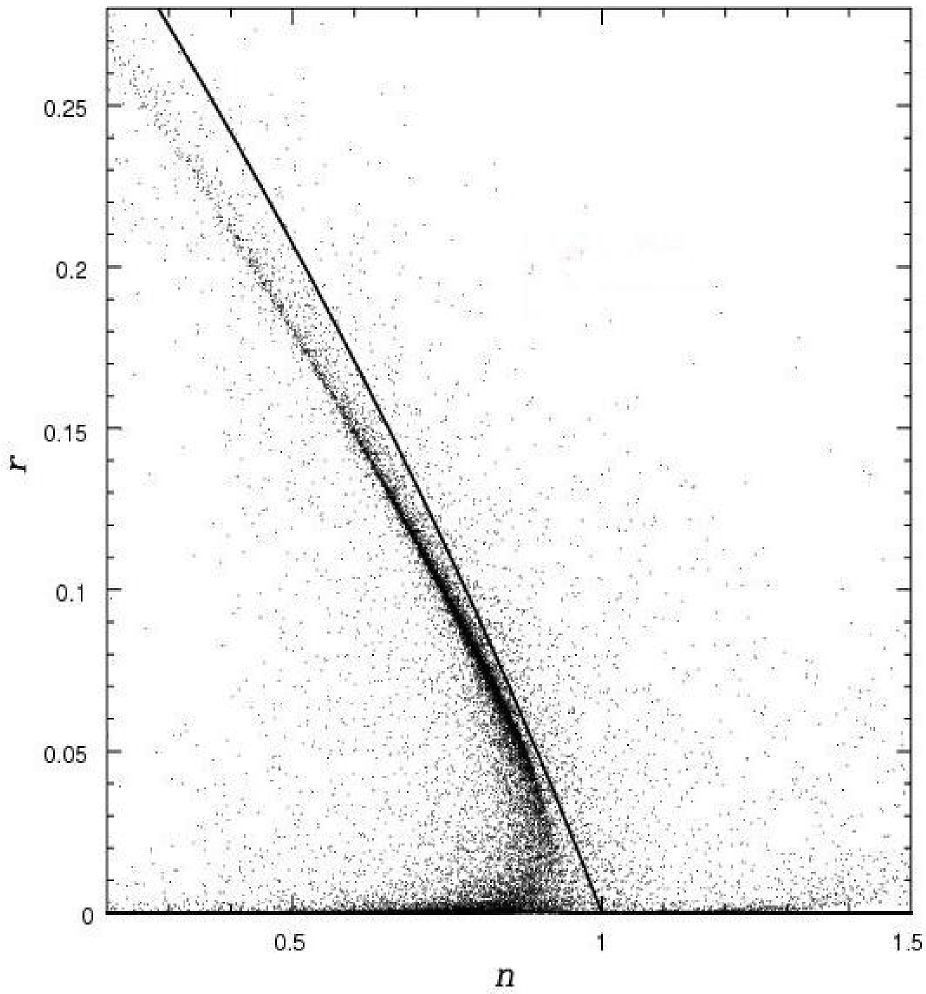
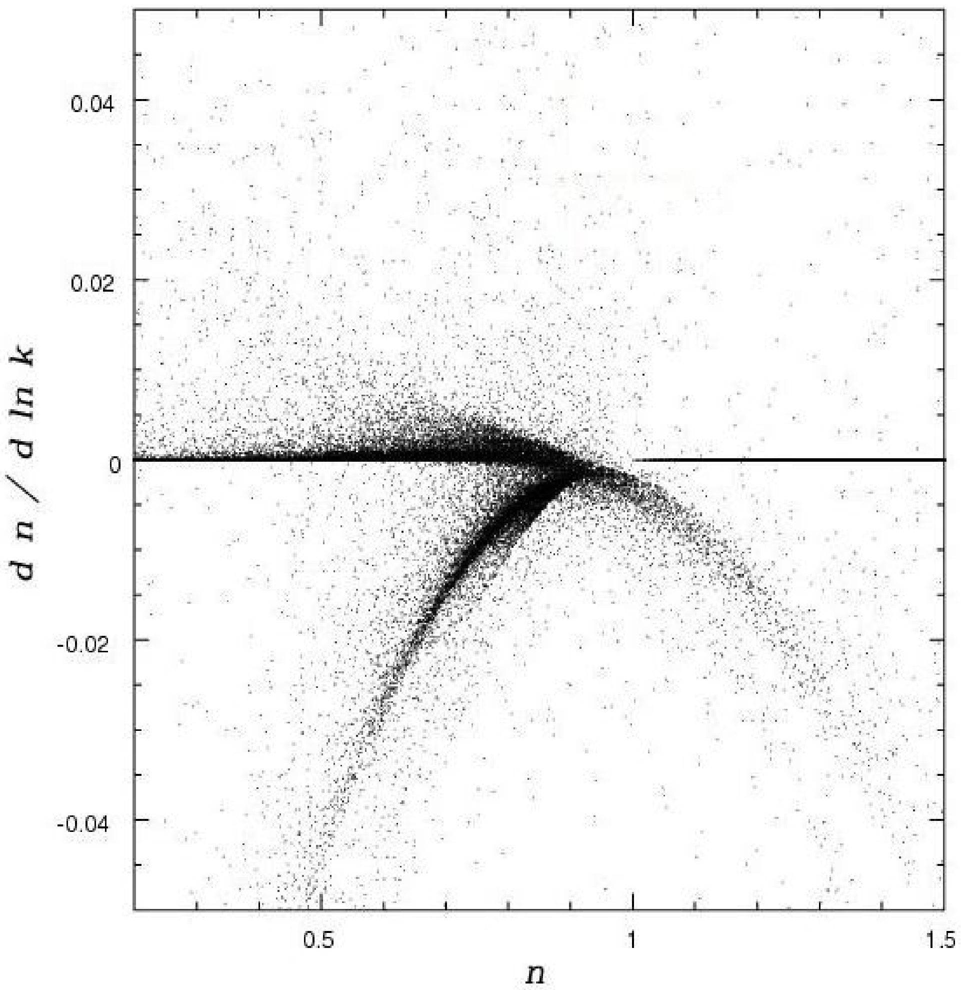
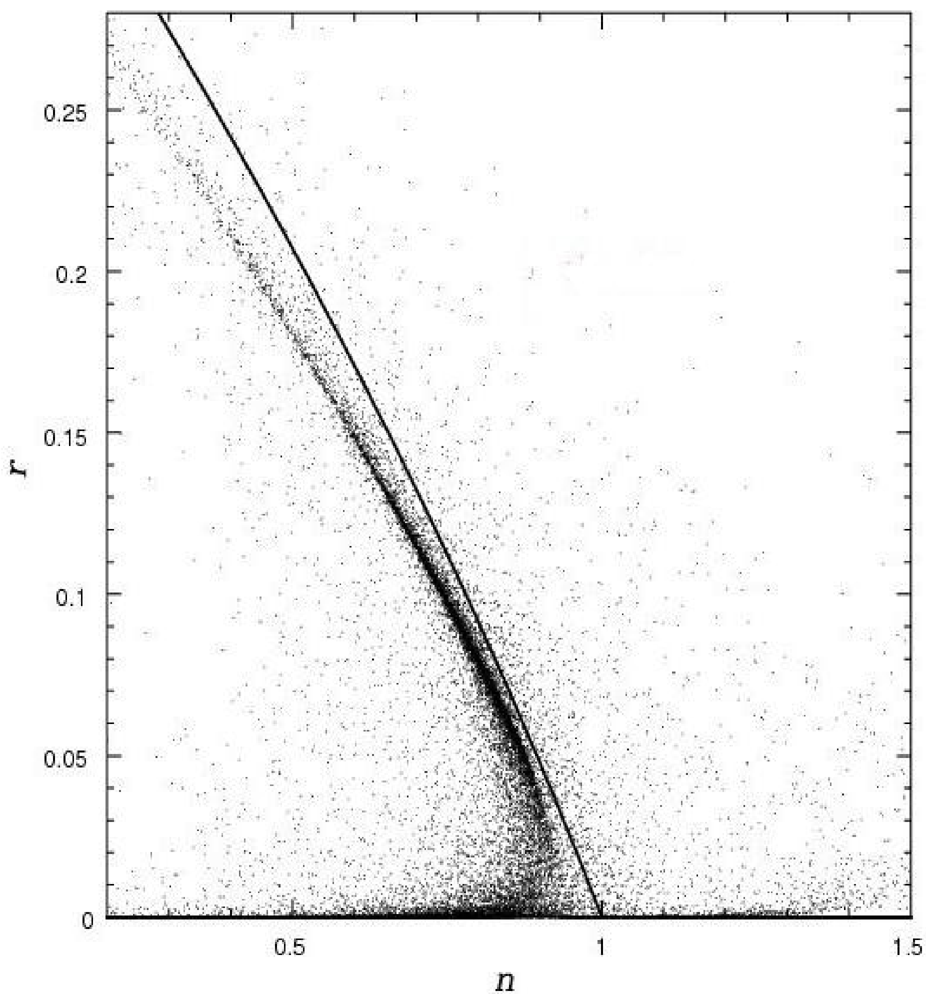
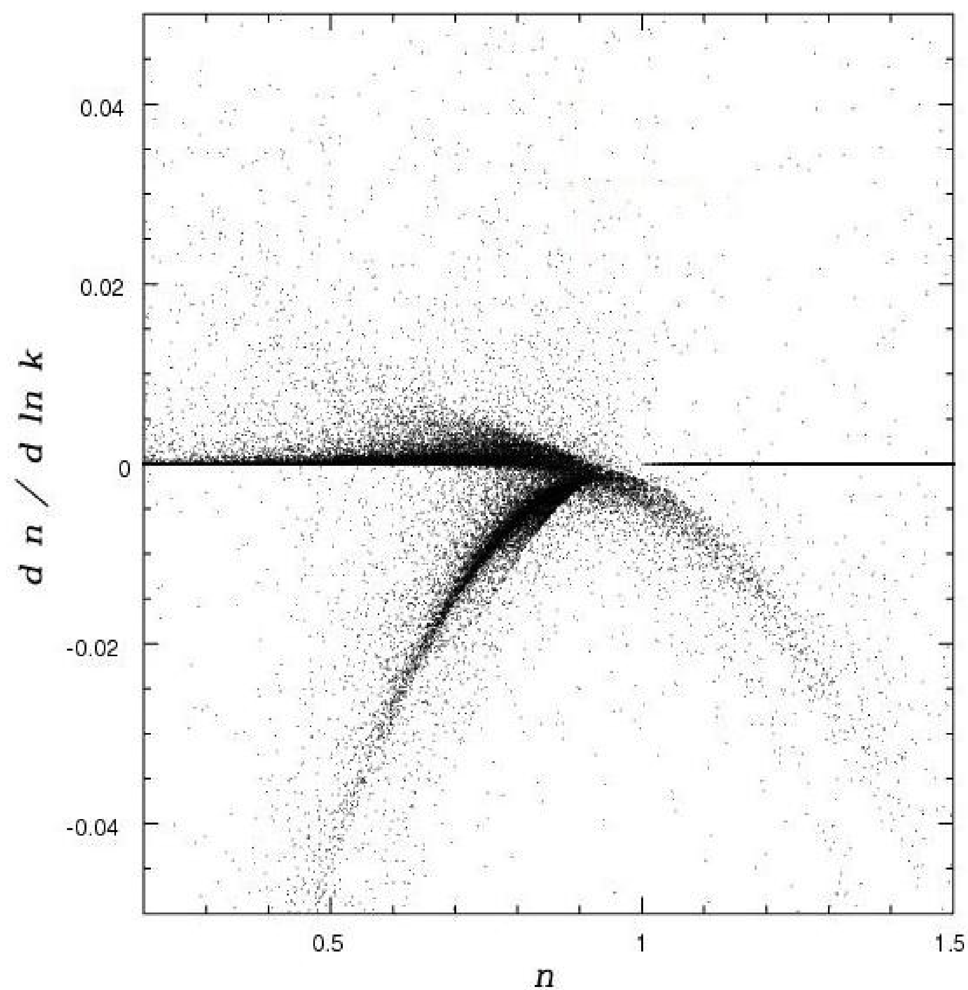




$$n_s(\epsilon, \eta, \xi^2, {}^3\lambda_H)$$

Flow Formalism





$n_s(\epsilon, \eta, \xi^2, \ ^3\lambda_H)$

$$\ln P(k) = (n_s - 1)\ln k + \frac{1}{2}\frac{dn_s}{d\ln k}\ln k^2$$

Evolution of Perturbations

- Inflaton fluctuations couple to gravity. Therefore, we must perturb both the gravity side and matter side of Einstein's Equations:

$$\delta G_{\mu\nu} = \frac{8\pi}{m_{\rm Pl}^2}\delta T_{\mu\nu}$$

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$\delta g_{\mu\nu}$

$\delta\phi$

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$\mathcal{R}$ → $\delta G_{\mu\nu}$ $\qquad$ $\delta\phi$ → $\delta T_{\mu\nu}$

- Next introduce the gauge invariant perturbation $u = a\delta\phi + z\mathcal{R}$,

$$u'' - \nabla^2 u - \frac{{z''}^2}{z}\, u = 0$$

$$z = \frac{a\dot{\phi}}{H}$$

Evolution of Perturbations

- Decompose:

$$u = \int \frac{d^3k}{(2\pi)^{3/2}} \left(v_k(\tau)\hat{a}_k(t)e^{ikx} + v_k^*(\tau)\hat{a}_k^\dagger(\tau)e^{-ikx} \right)$$

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Cosmological dynamics

Calculating the Spectrum

- No general analytic solution.
- However, in the short wavelength limit, $k \gg aH$, we recover the quasi-Minkowski wavefunction,

$$v_k'' + k^2 \left(1 - 2\left(\frac{aH}{k}\right)^2 F(\epsilon, \eta, \xi^2)\right) v_k = 0$$

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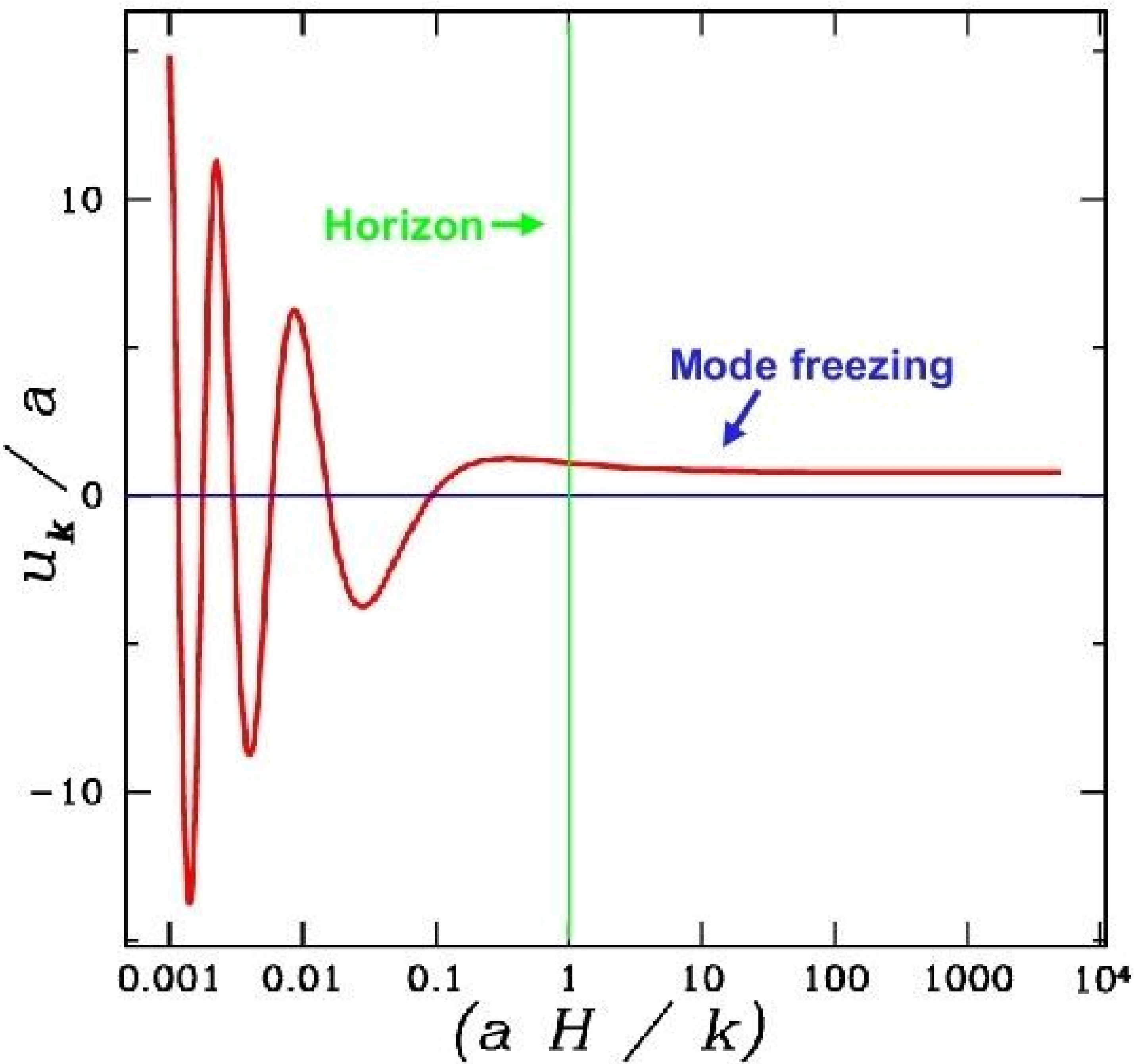
Initial Conditions

- Example: slow roll inflation,

$$v_k \propto \sqrt{-k\tau} H_\nu(-k\tau)$$



Horizon
Mode freezing
10
0
−10
0.001
0.01
0.1
1
10
100
1000
10^4
u_k / a
$(a\ H / k)$

Calculating the Spectrum

- Having solved the mode equation, we compute the correlation function,

$$\langle \delta\phi(x)\delta\phi(y)\rangle = \frac{1}{2\pi^3}\int |v_k|^2 e^{-ik(x-y)} d^3k$$

- We also define the *power spectrum*

$$\langle \delta\phi(x)\delta\phi(y)\rangle = \int d\mathrm{ln}k\, P(k) e^{-ik(x-y)}$$

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$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}\left|\frac{v_k}{z}\right|^2$$

Strategy

- We will obtain an inflationary solution using the flow method, $H(\phi)$.

- Given this solution, we numerically evolve the mode equation,

$$v_k'' + k^2 \left(1 - 2\left(\frac{aH}{k}\right)^2 F(\epsilon, \eta, \xi^2)\right) v_k = 0$$

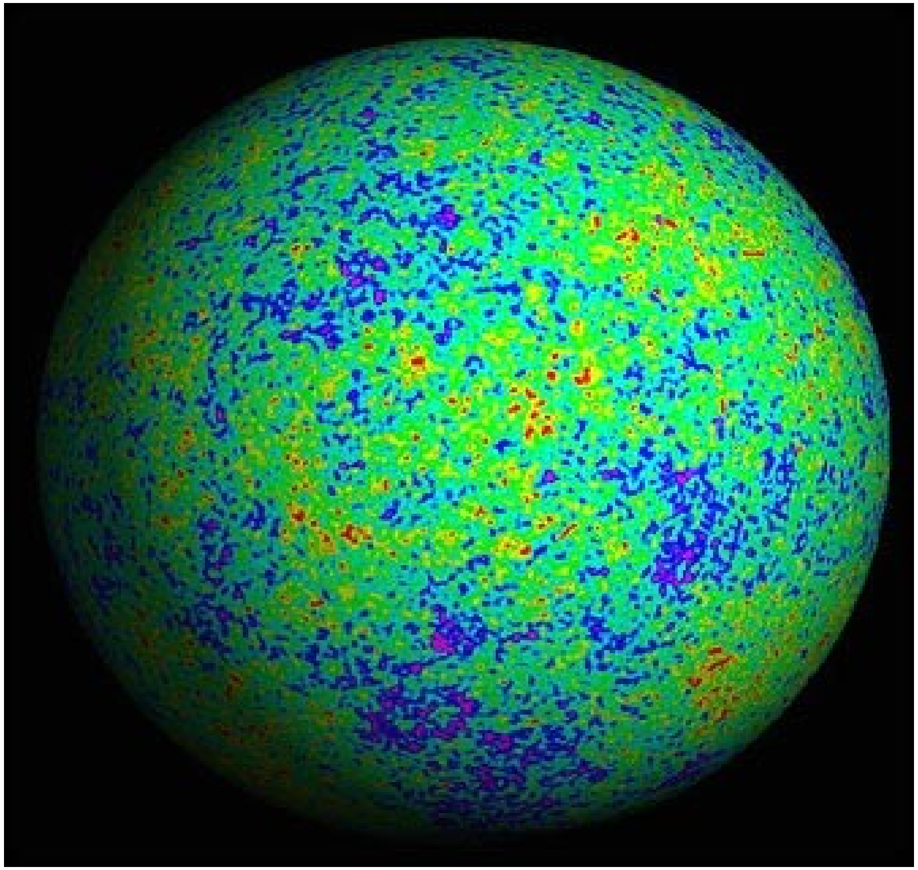
- We do this for a couple thousand k's, and build the power spectrum,

$$P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \left|\frac{v_k}{z}\right|^2$$

- We then want to compare this power spectrum with current data.

CMB

- The Universe's baby picture.



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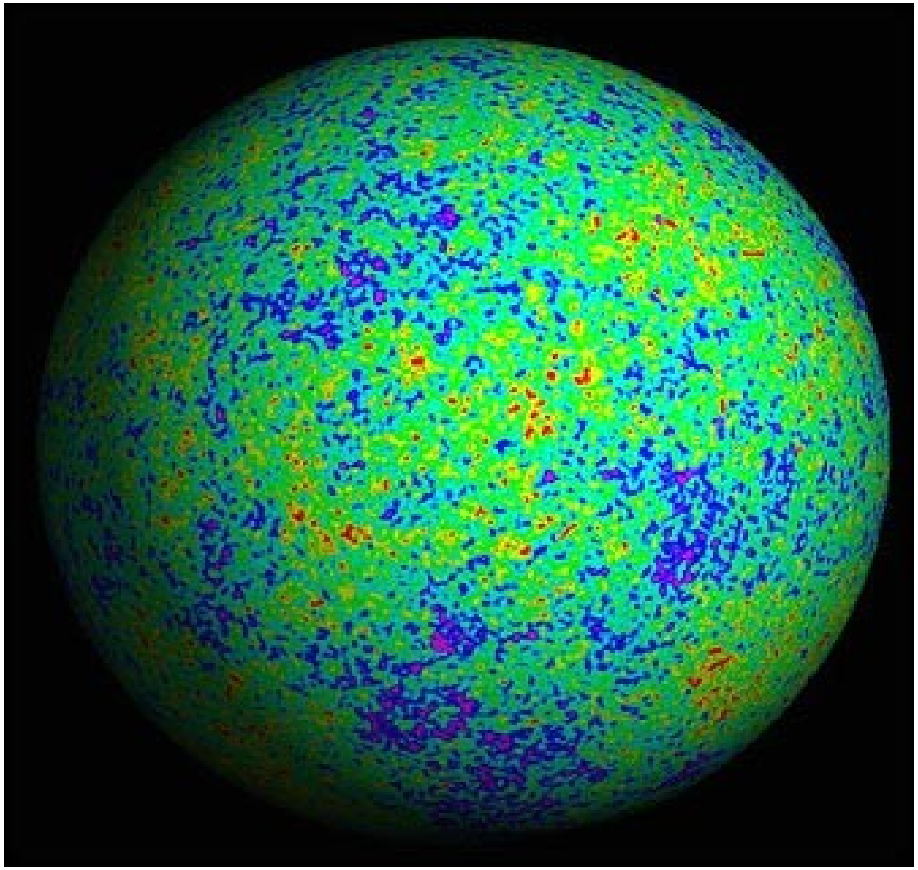
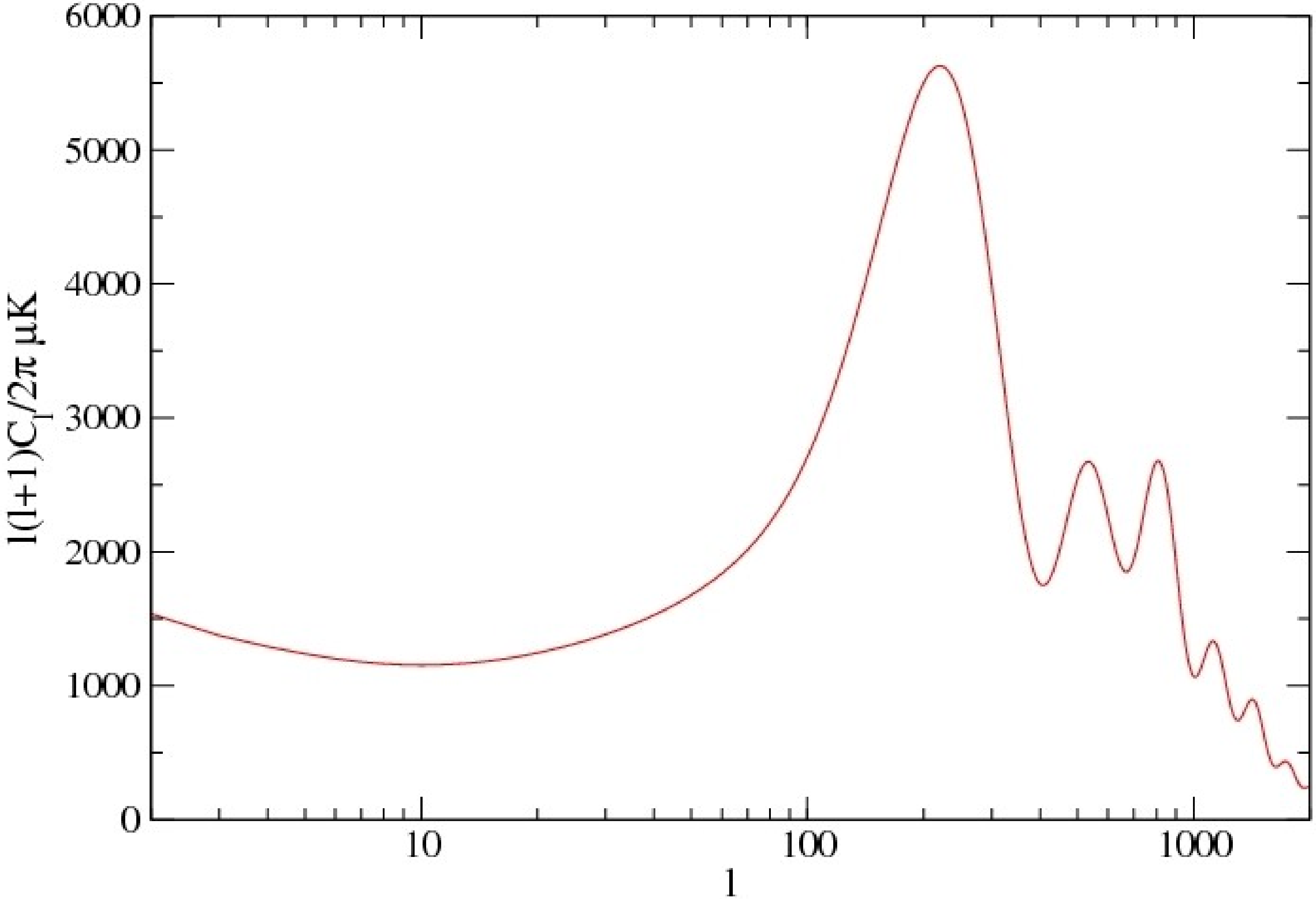


$$\frac{\delta T}{T}(\vec{n}) = \sum_{\ell,m} a_{\ell m} Y_{\ell m}(\vec{n})$$

$$C(\theta) = \left\langle \frac{\delta T}{T}(\vec{n_1}) \frac{\delta T}{T}(\vec{n_2}) \right\rangle$$

CMB



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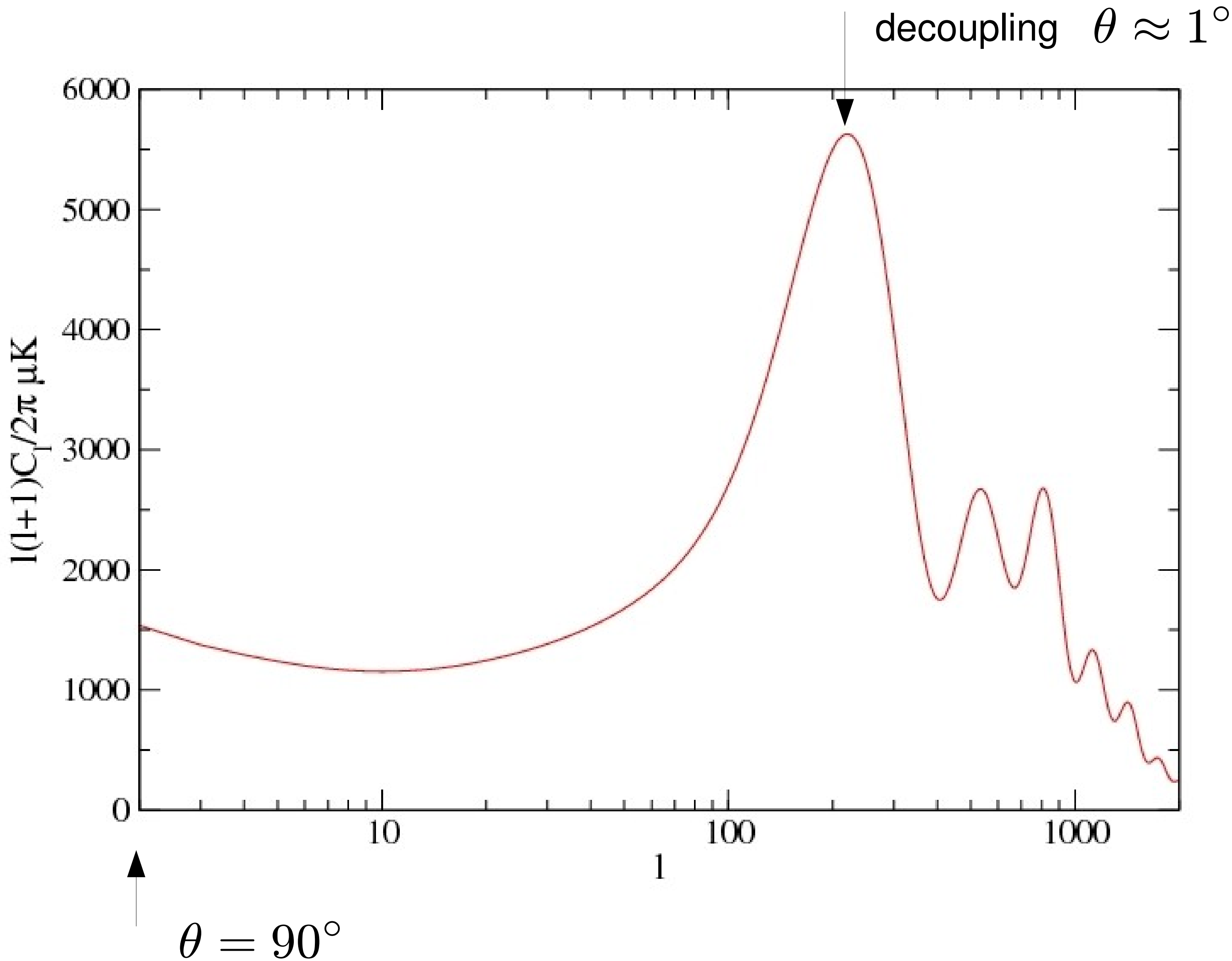
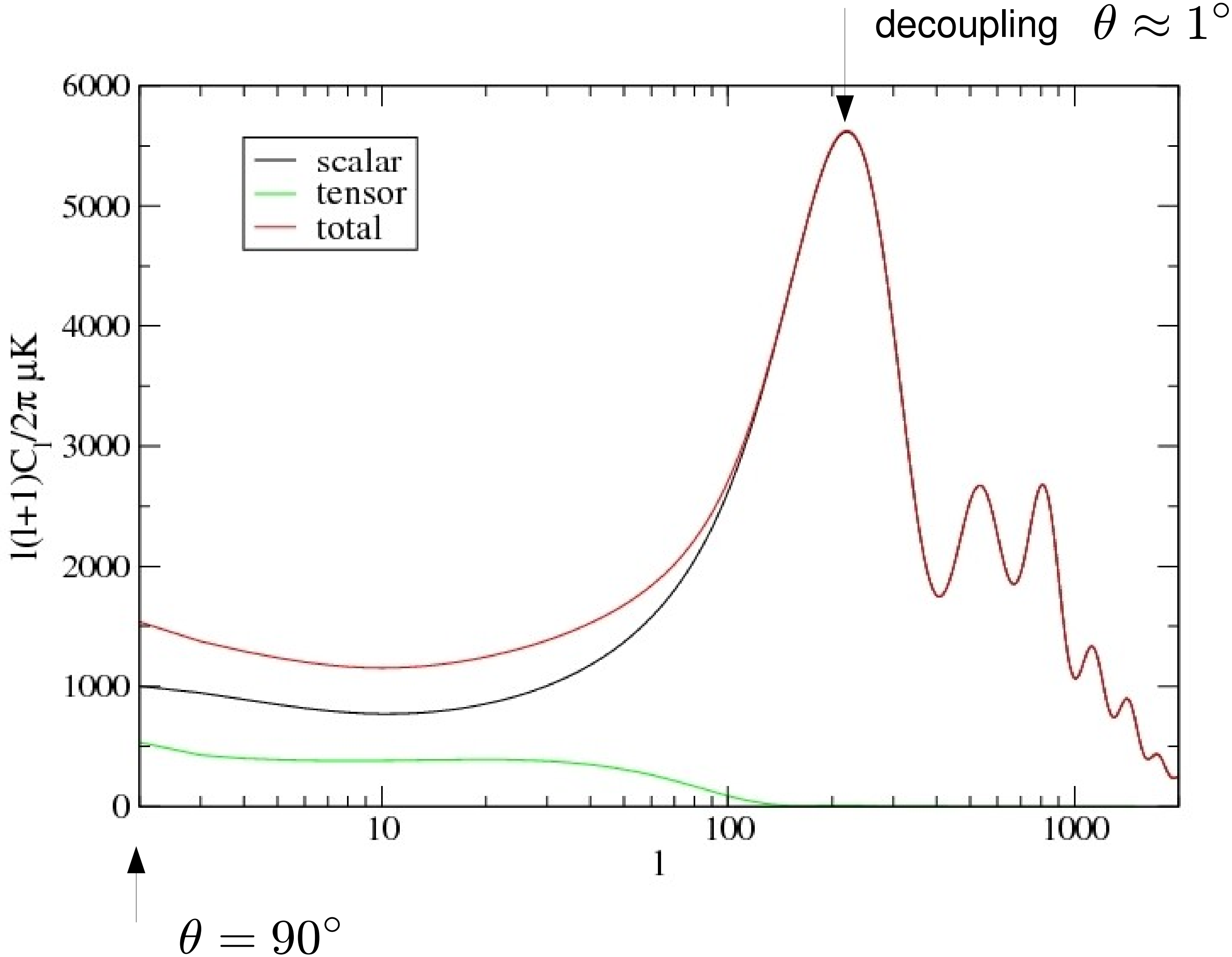
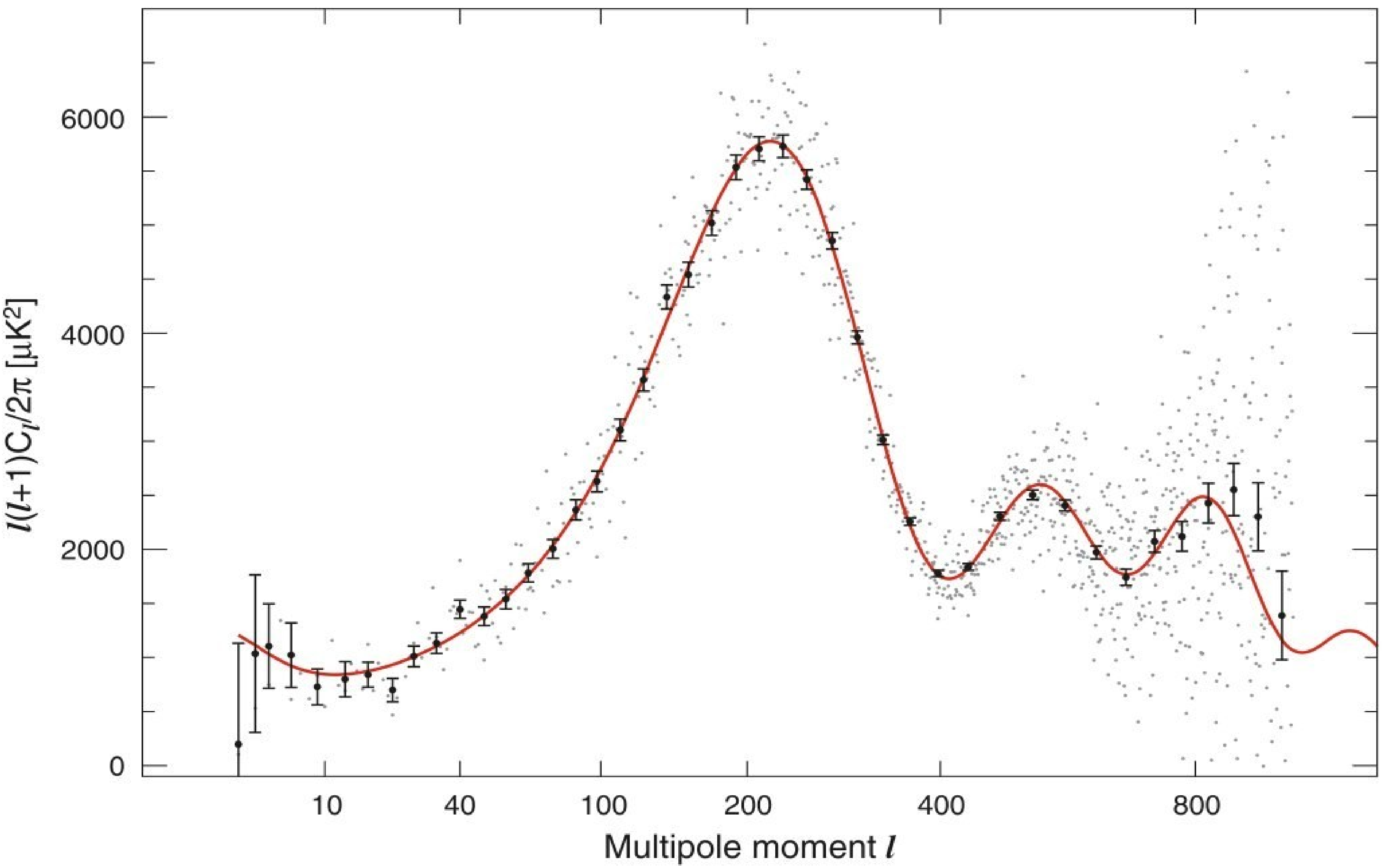


decoupling $\theta \approx 1^\circ$
6000
5000
4000
3000
2000
1000
0
10
100
1000
l
l(l+1)C_l/2π μK
$\theta = 90^\circ$

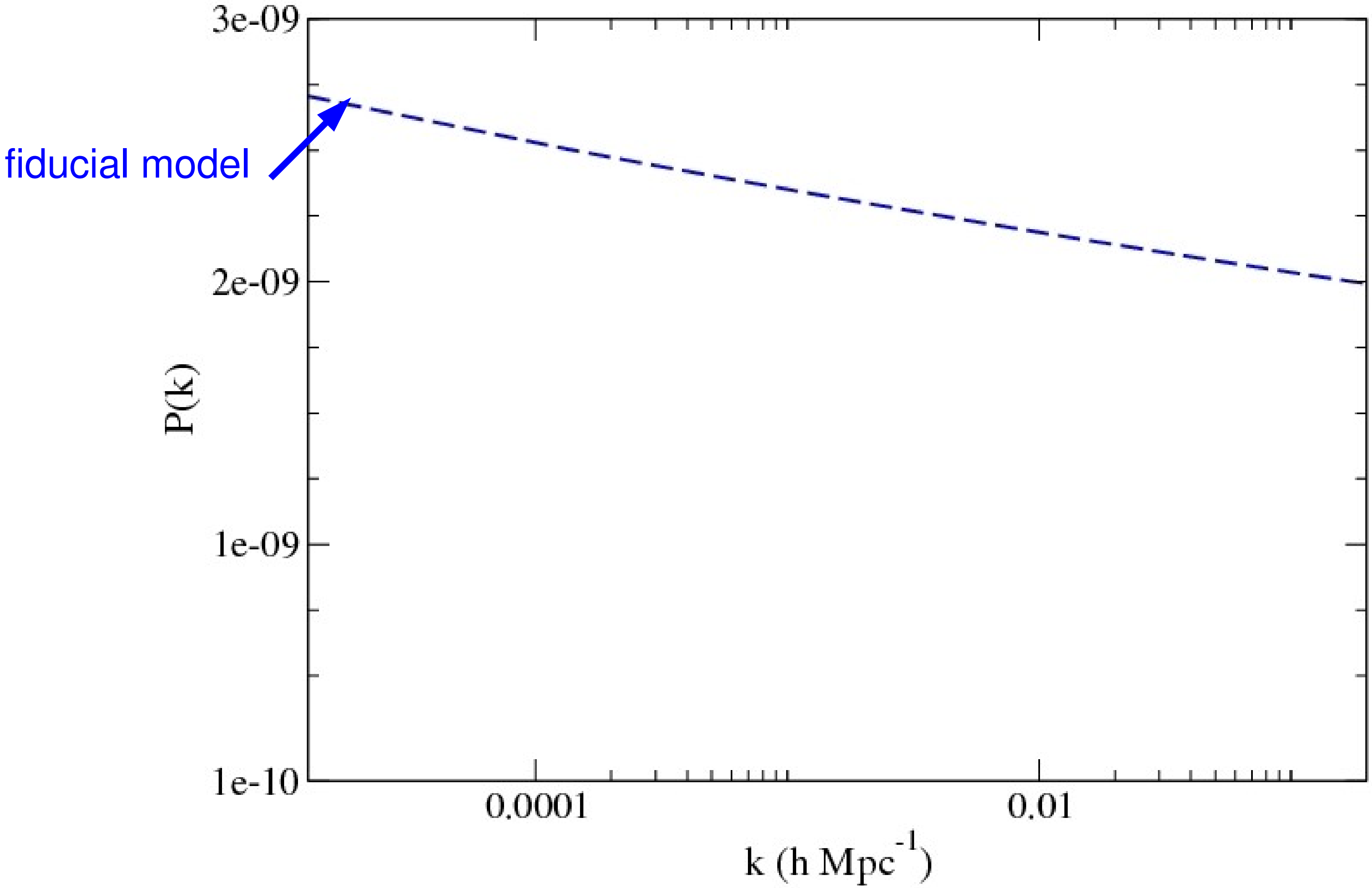
CMB


decoupling $\theta \approx 1^\circ$
6000
5000
4000
3000
2000
1000
0
scalar
tensor
total
l(l+1)C_l/2π μK
10
100
1000
l
$\theta = 90^\circ$

WMAP5 TT Spectrum



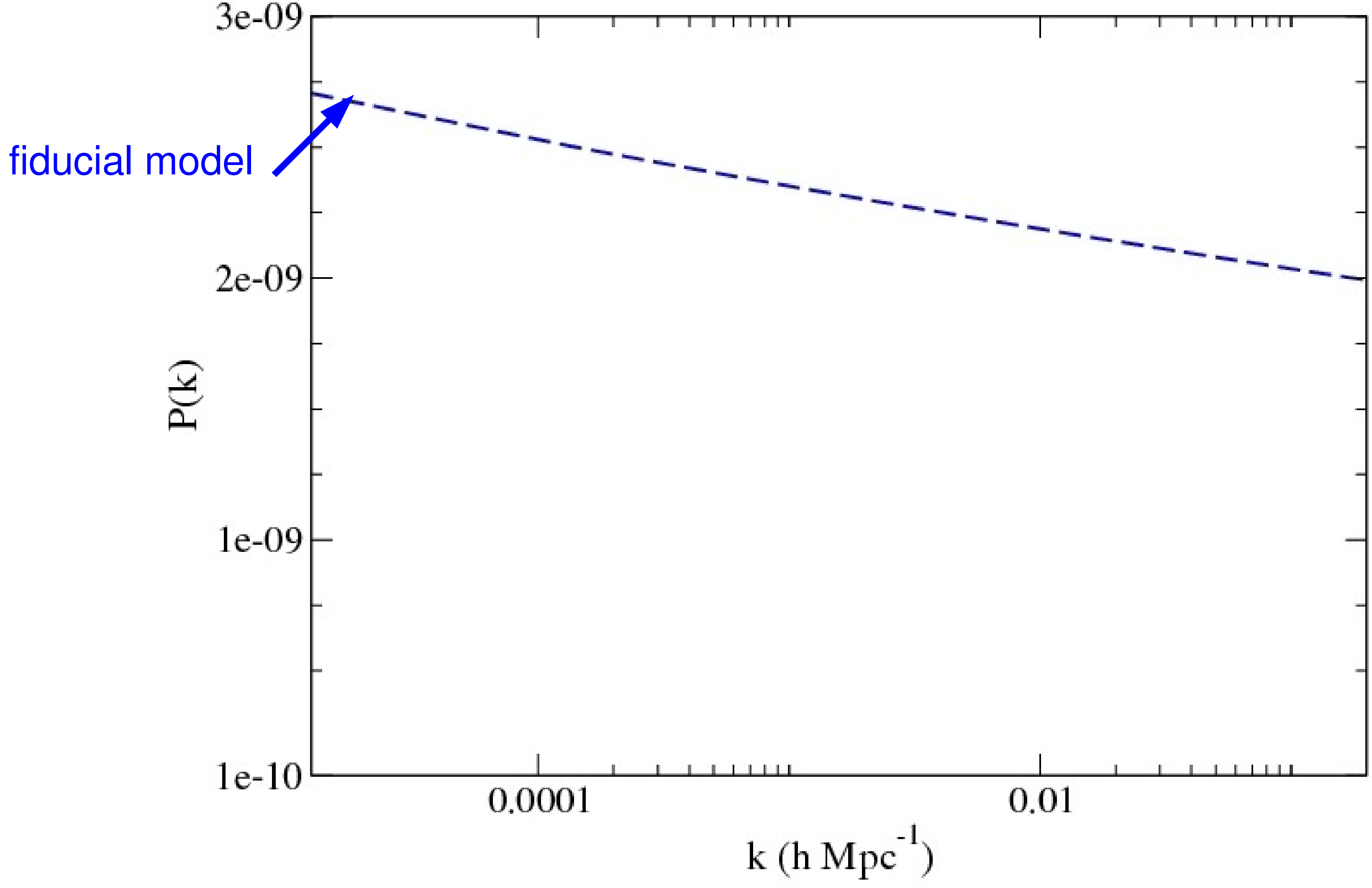
Results



Results

$$p_\nu = \int_{\chi^2}^{\infty} P_\nu(y)dy$$

$$|p_{\rm trial} - p_{\rm fid}| \leq 0.01$$



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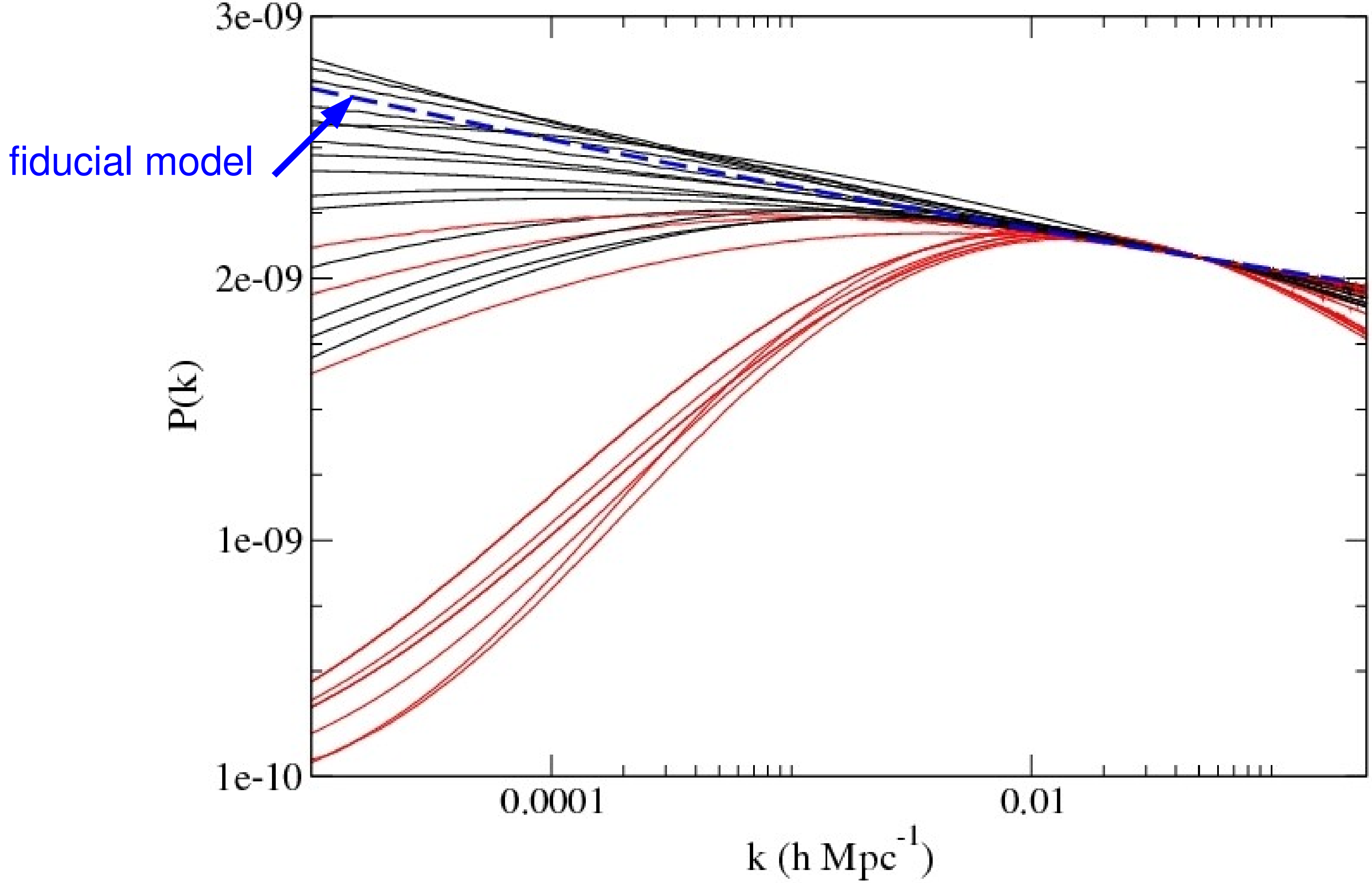

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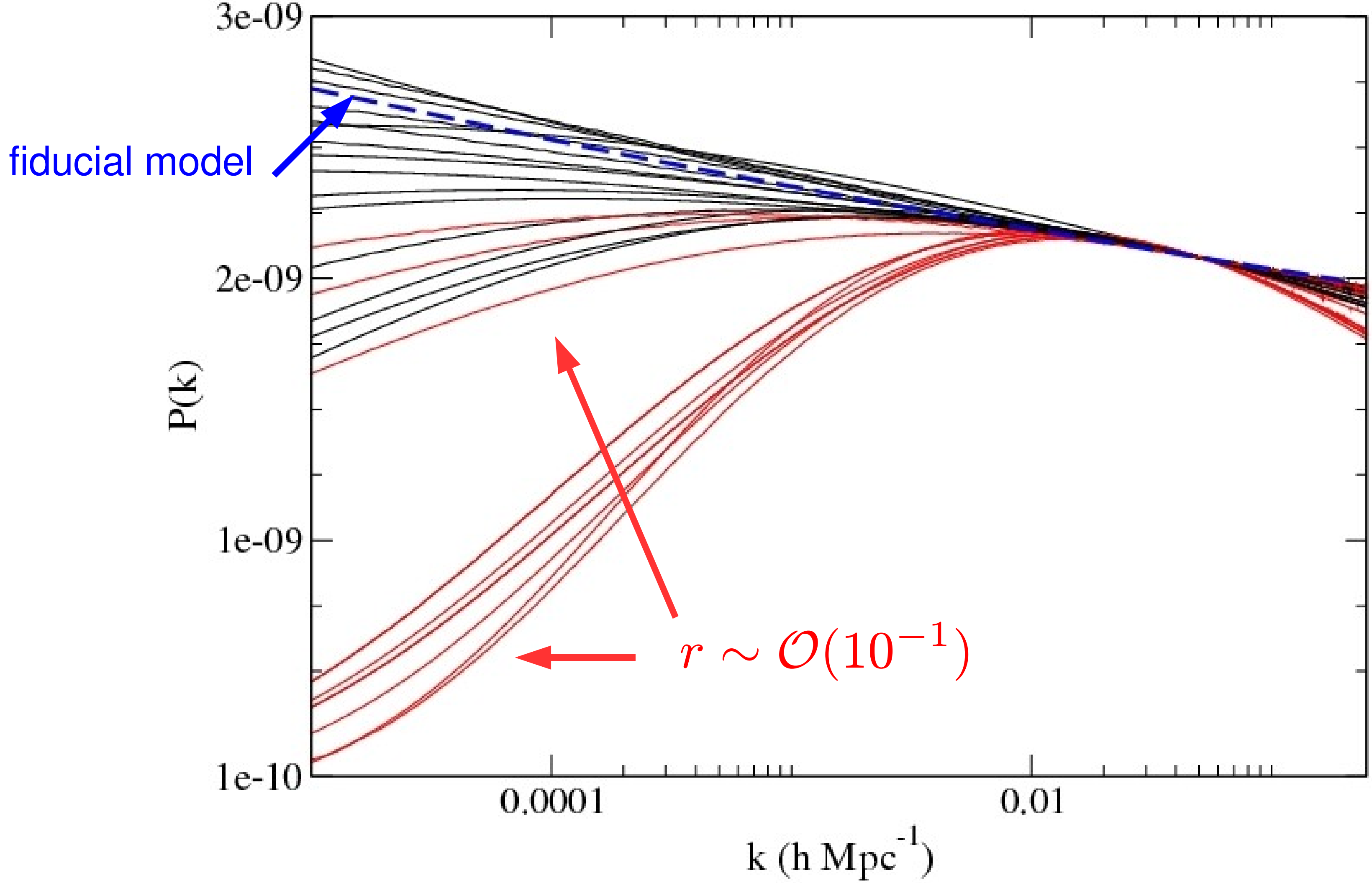
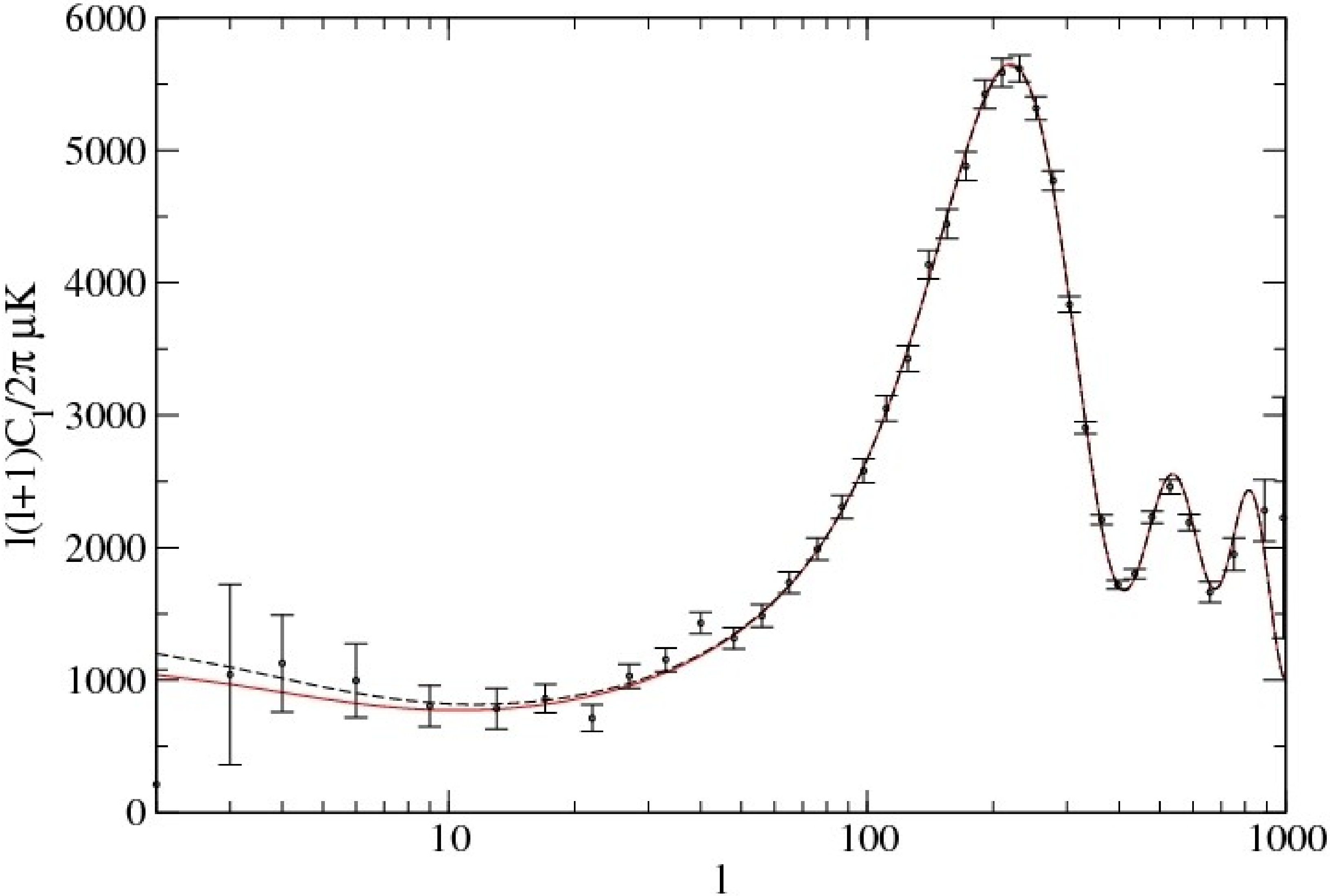
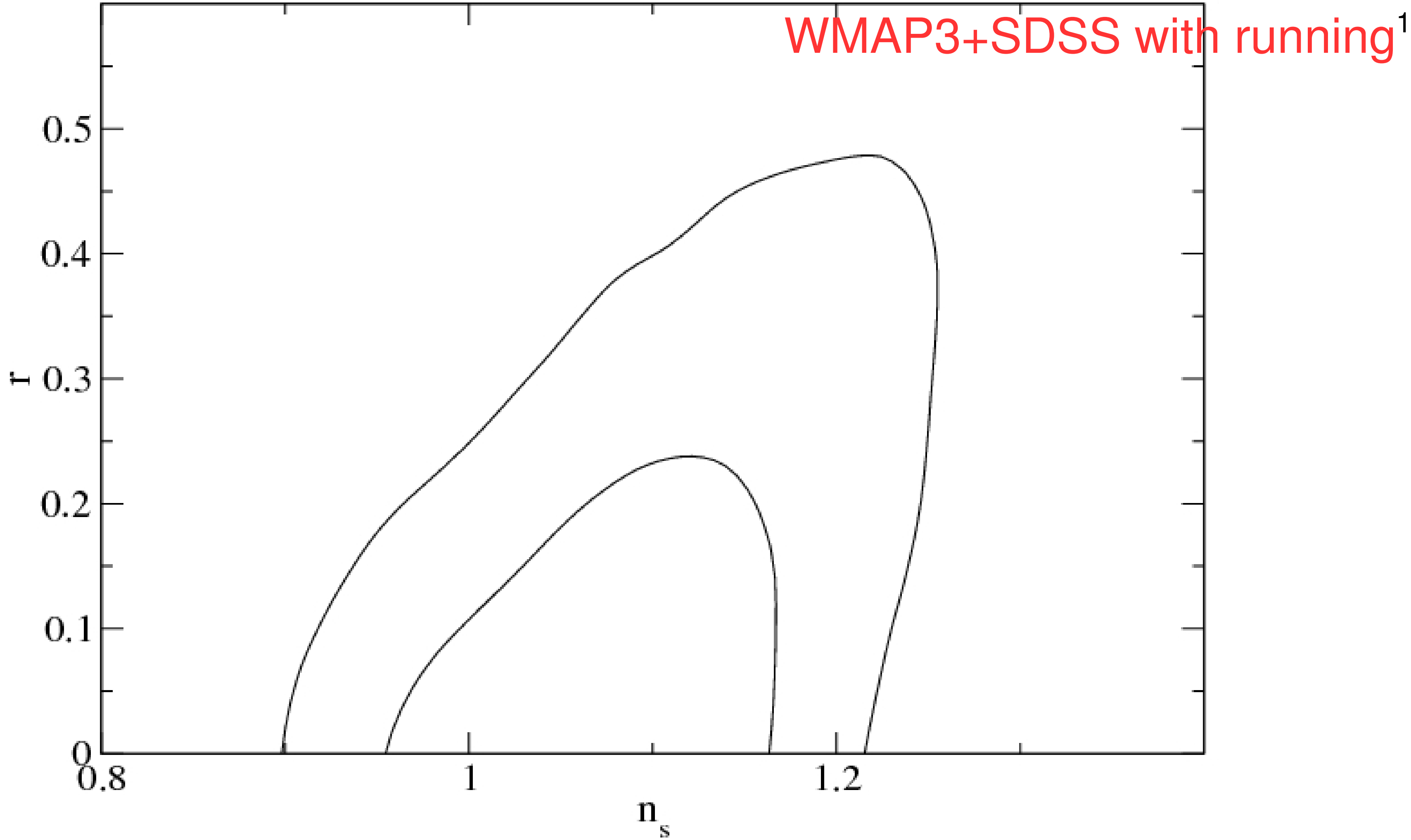

fiducial model
$r \sim \mathcal{O}(10^{-1})$
3e-09
2e-09
1e-09
1e-10
P(k)
0.0001
0.01
k (h Mpc^{-1})

CMB Spectra



Comparison with Previous Studies



[1]Kinney, Kolb, Melchiori, Riotto astro-ph/0605338

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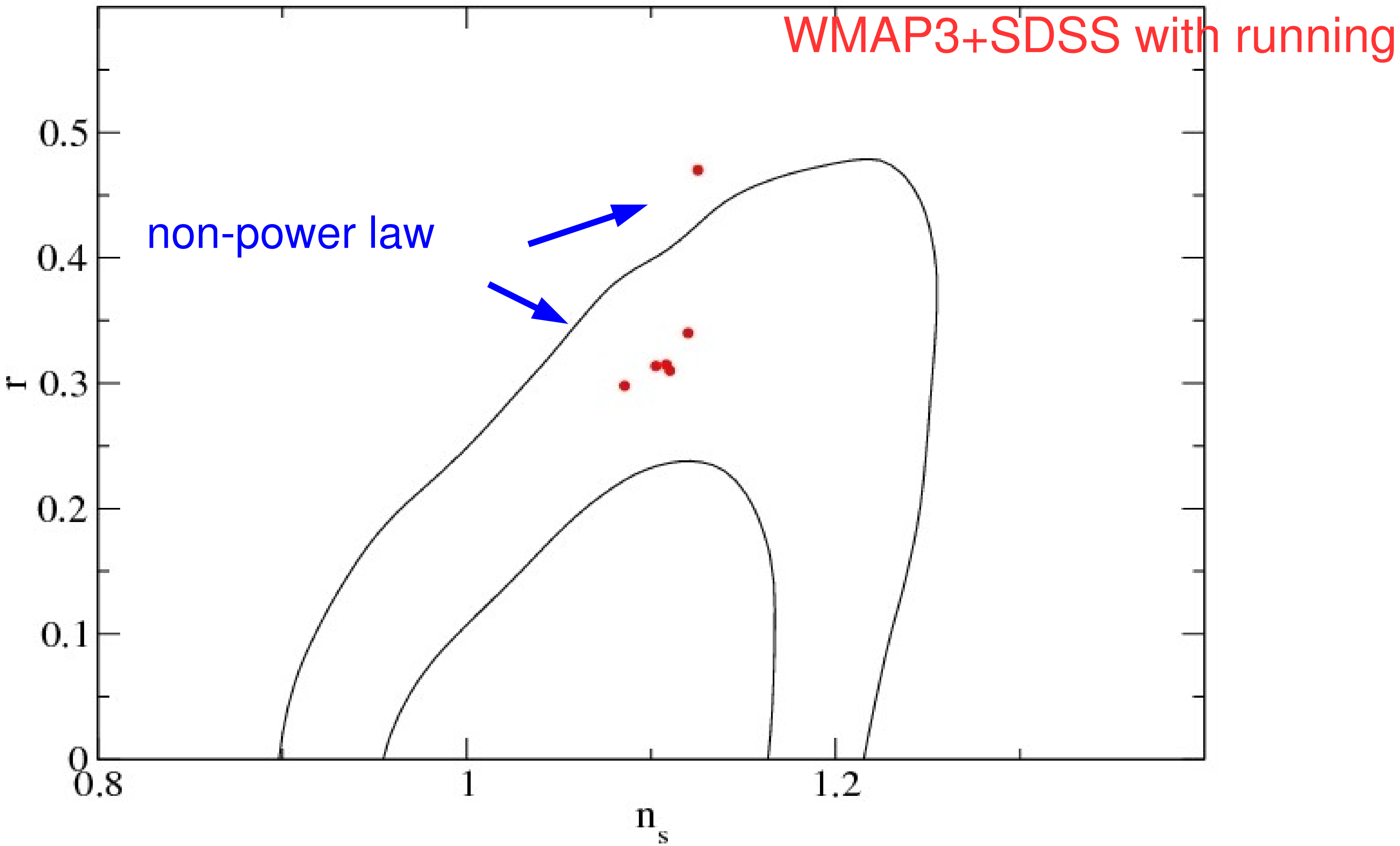

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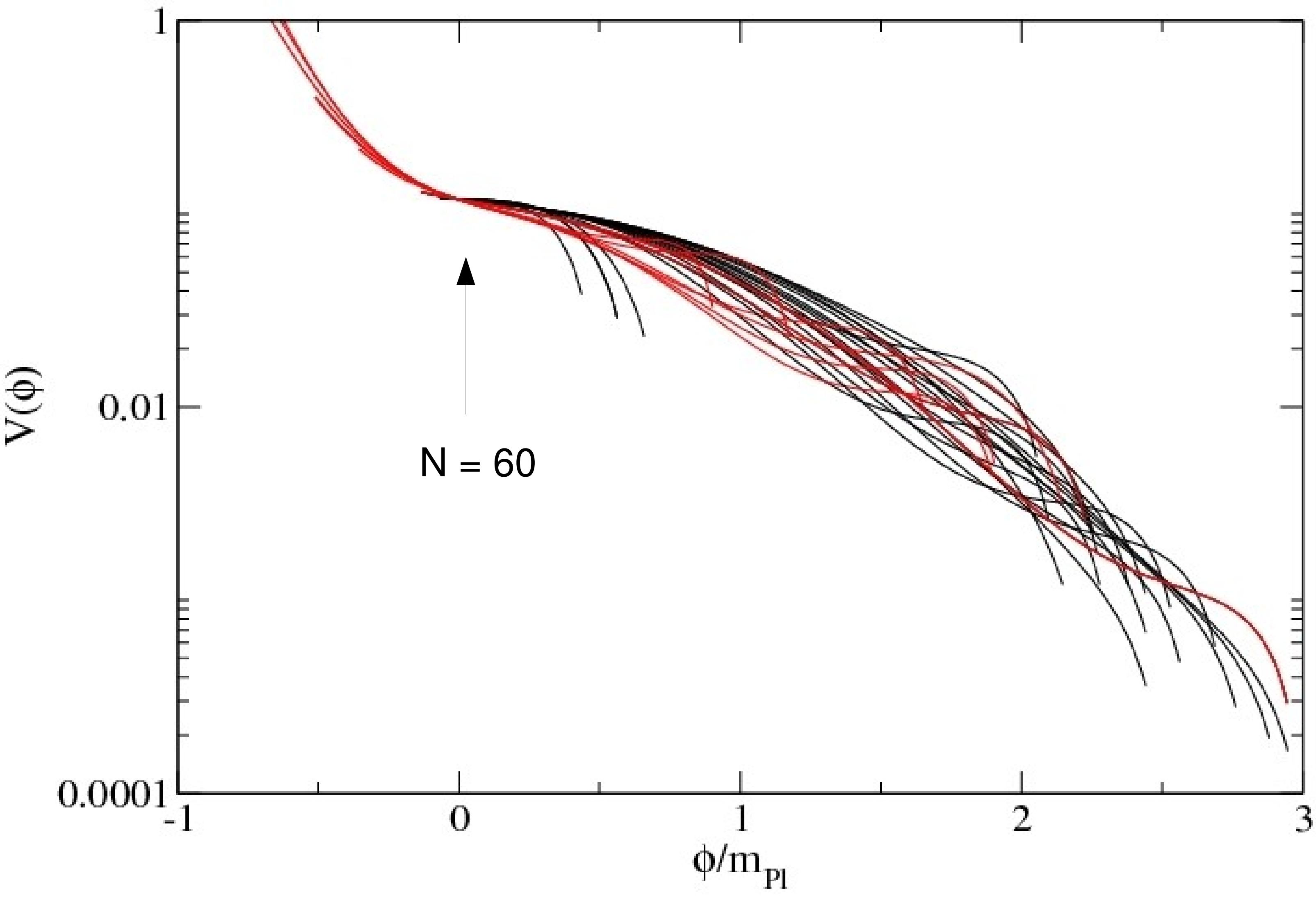
Inflaton Potentials



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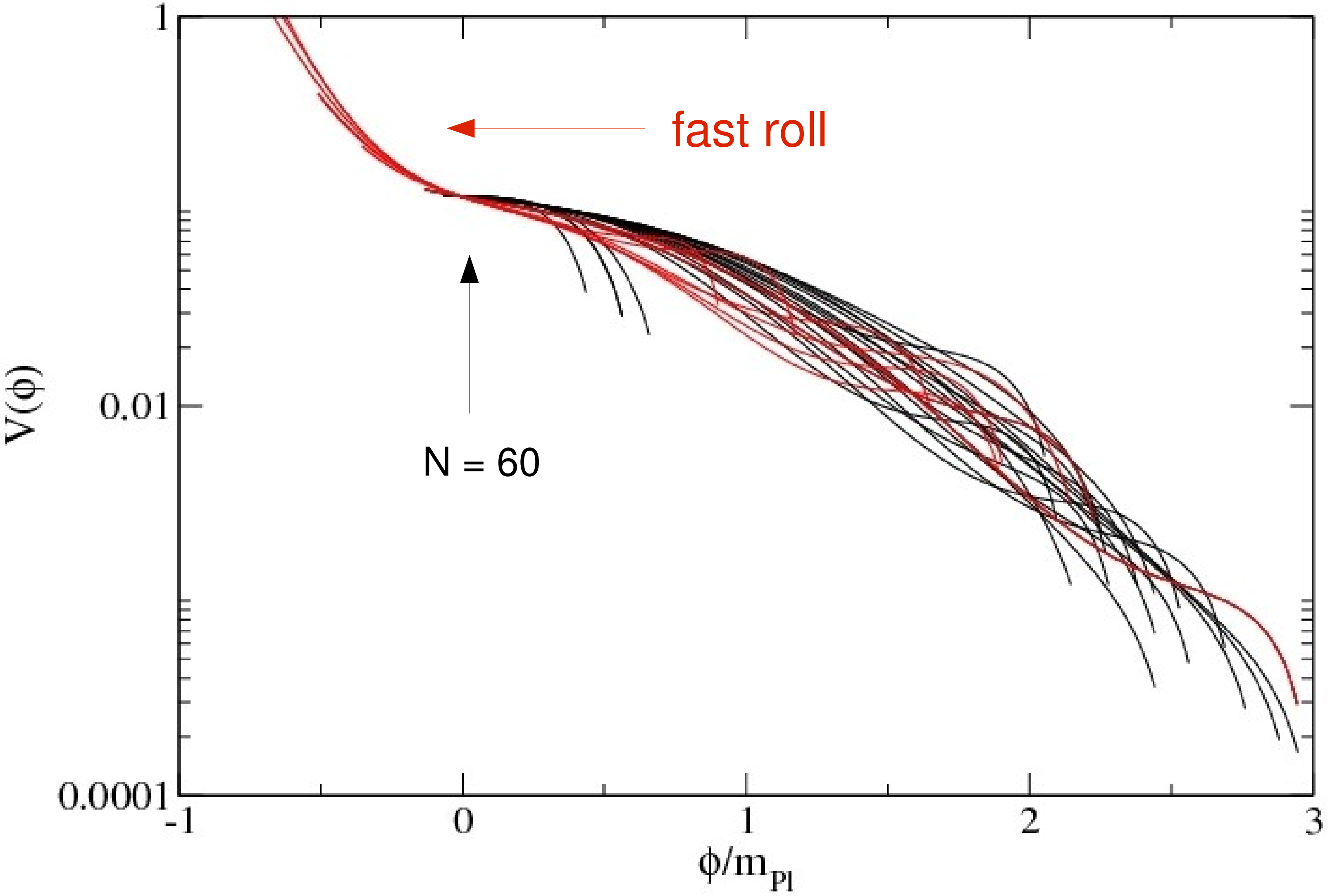

Summary

- We have developed a method useful for generating a wide range of inflation models that are consistent with current cosmological data.
- Cosmic variance, existing at large scales, is the dominant source of error in CMB maps.
- This translates to an uncertainty in the form of the primordial power spectrum on large scales.
- Which translates to an uncertainty in the initial dynamics of the inflaton field.
- Our results suggest that fast rolling fields might be considered equally consistent with current data as the well studied slow roll models.

Non-canonical Inflation

- Consider an action with a generalized kinetic term:

$$S = \frac{1}{2}\int d^4x \left[M_{\rm Pl}^2 R + 2\mathcal{L}\left(\frac{1}{2}\dot{\phi}^2, \phi\right)\right]$$

- Such forms arise from effective field theories, e.g. brane dynamics.

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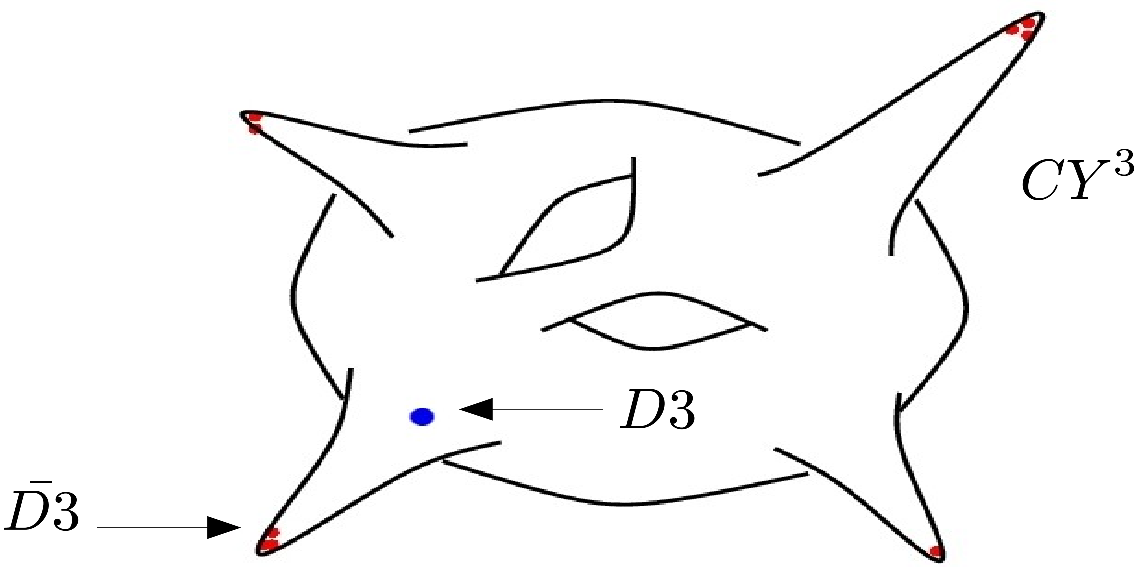

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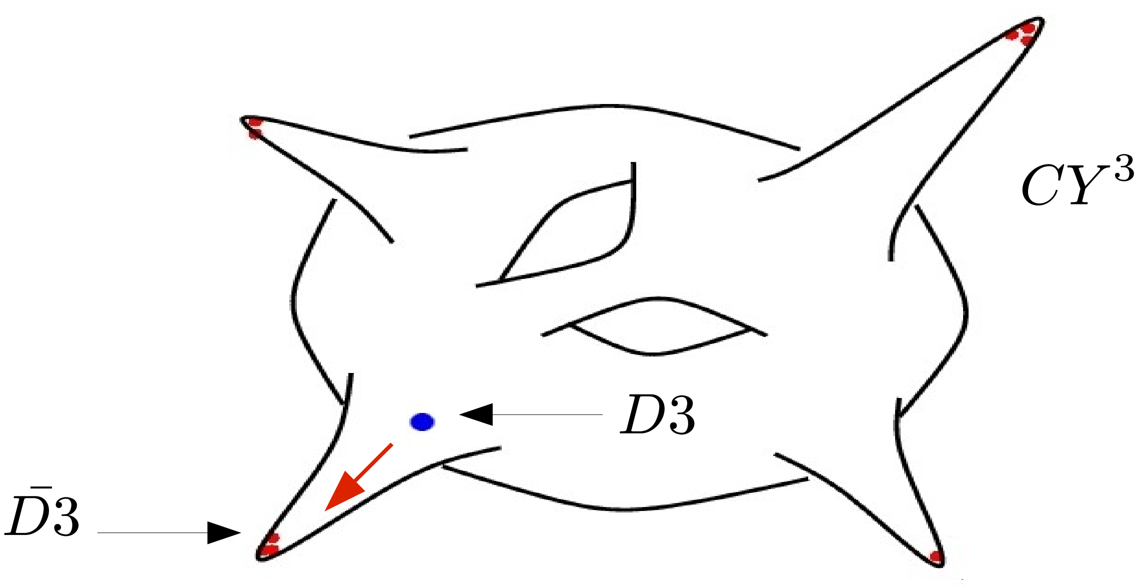

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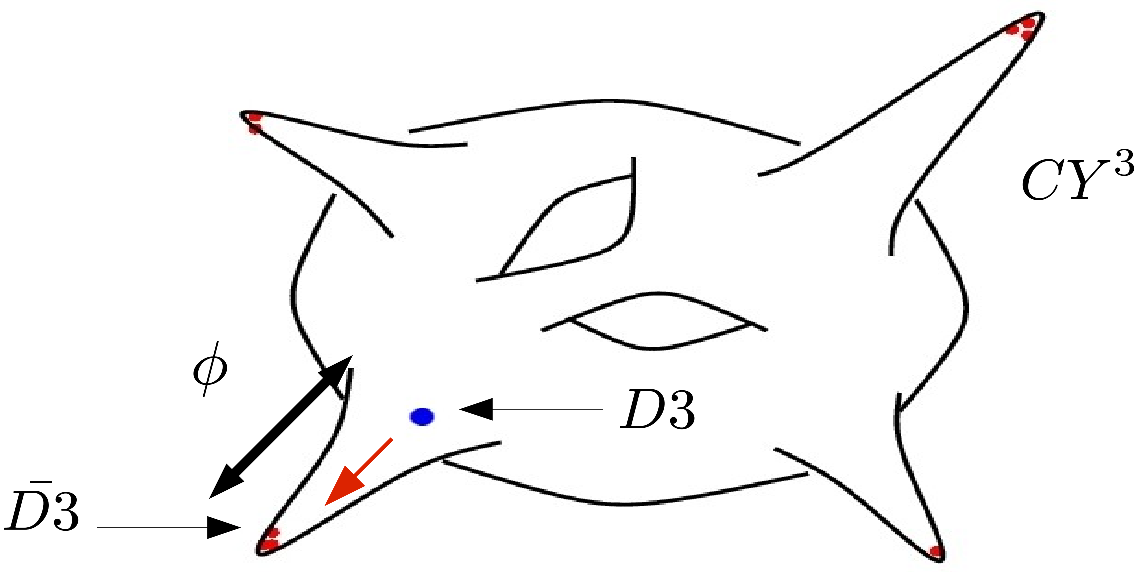

DBI Inflation

- Lagrangian:

$$\mathcal{L} = -f^{-1}(\phi)\sqrt{1 - f(\phi)\dot{\phi}^2} + f^{-1}(\phi) - V(\phi)$$

$$f(\phi) = \frac{1}{T_3 h^4(\phi)}$$

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→ These fluctuations are non-Gaussian!

Observables

- Non-gaussianities:

$$\Phi(x) = \Phi_g(x) + f_{NL}(\Phi_g^2(x) - \langle \Phi_g^2(x) \rangle)$$

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$$\eta = 2M_{\rm Pl}^2 c_s \frac{H''}{H}$$

A Reconstruction Forecast

- In the best of all worlds, we will detect tensors *and* non-Gaussianties.

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- Question: How big of an impact will an unresolved c_s have on reconstruction?

A Reconstruction Forecast

- We generate simulated Planck-precision data: T, E-, and B-mode polarization out to $\ell = 2000$.

- We assume a failure to detect non-Gaussianities: $f_{NL} < |5|$

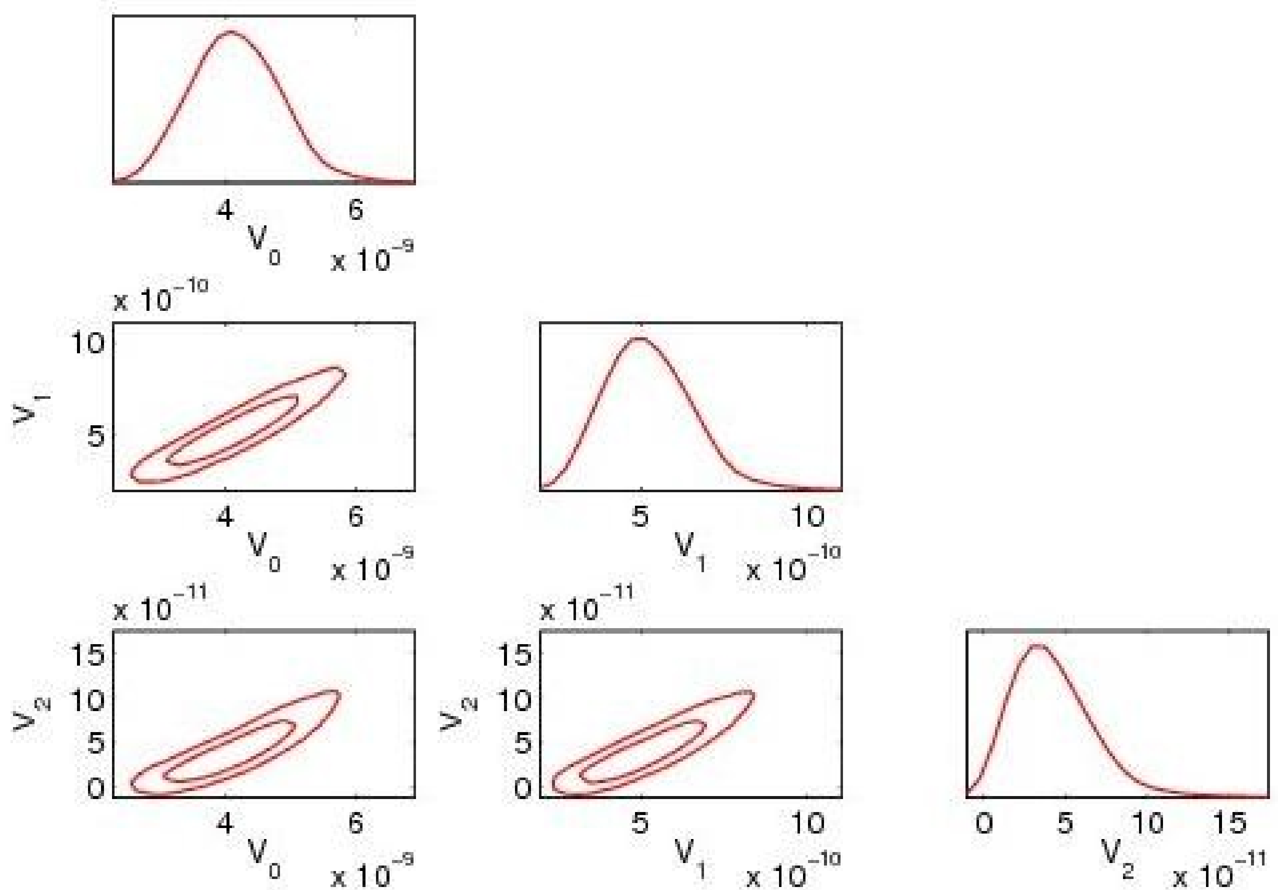
- We wish to perform Bayesian parameter estimation on the system:

$$V_0,\ V_0',\ V_0''$$

- This is accomplished via Markov Chain Monte Carlo, where we vary

  $V_0,\ V_0',\ V_0''$ directly in the chains.

Canonical Reconstruction



A Reconstruction Forecast

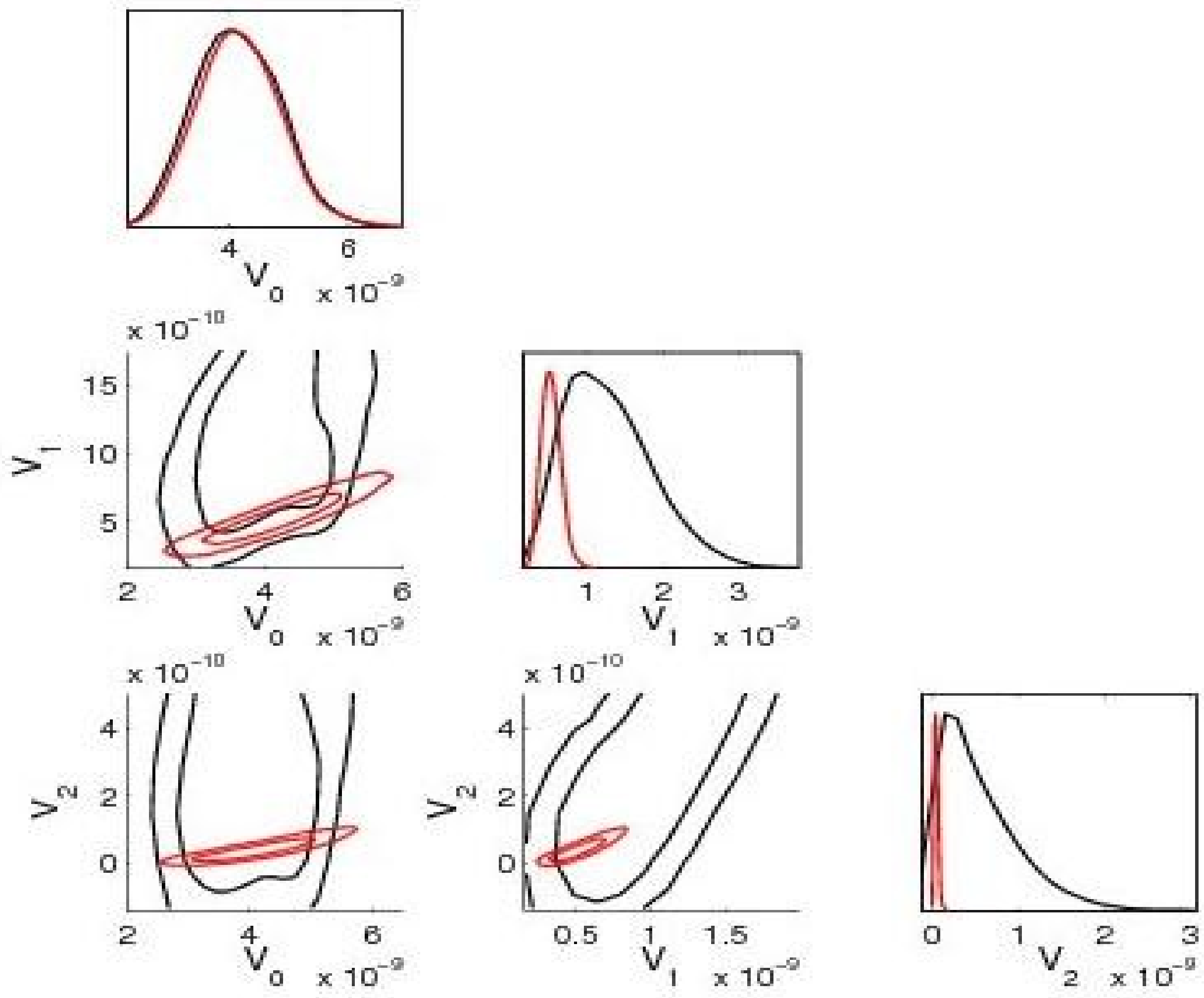
- A null detection of non-Gaussianities, $f_{NL} < |5|$, for DBI inflation means $c_s \in [0.25, 1]$.

- To reconstruct non-canonical inflation, we need to consider the larger system:

$$V_0,\ V_0',\ V_0'',\ c_s$$

where c_s is allowed to vary in the chains within the above prior range.

Non-canonical Reconstruction



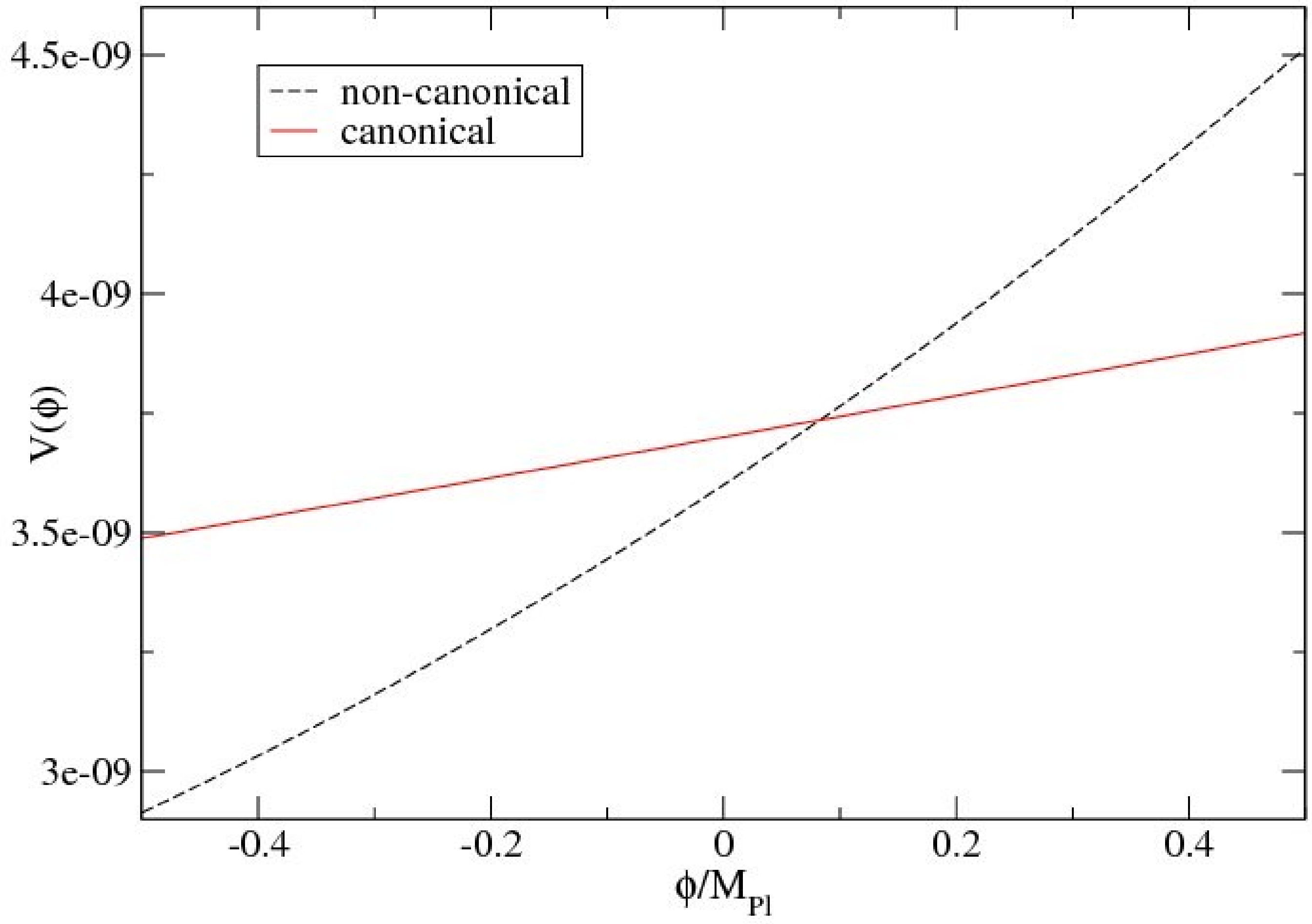
Marginalized Errors

	$V_0 \times 10^9 M_{\rm Pl}^4$	$V_0' \times 10^{10} M_{\rm Pl}^3$	$V_0'' \times 10^{11} M_{\rm Pl}^2$
canonical	$3.7^{+1.9}_{-0.7}$	$4.3^{+3.7}_{-1.2}$	$2.5^{+7.2}_{-1.8}$
non-canonical	$3.6^{+1.9}_{-0.9}$	16^{+10}_{-12}	91^{+80}_{-90}

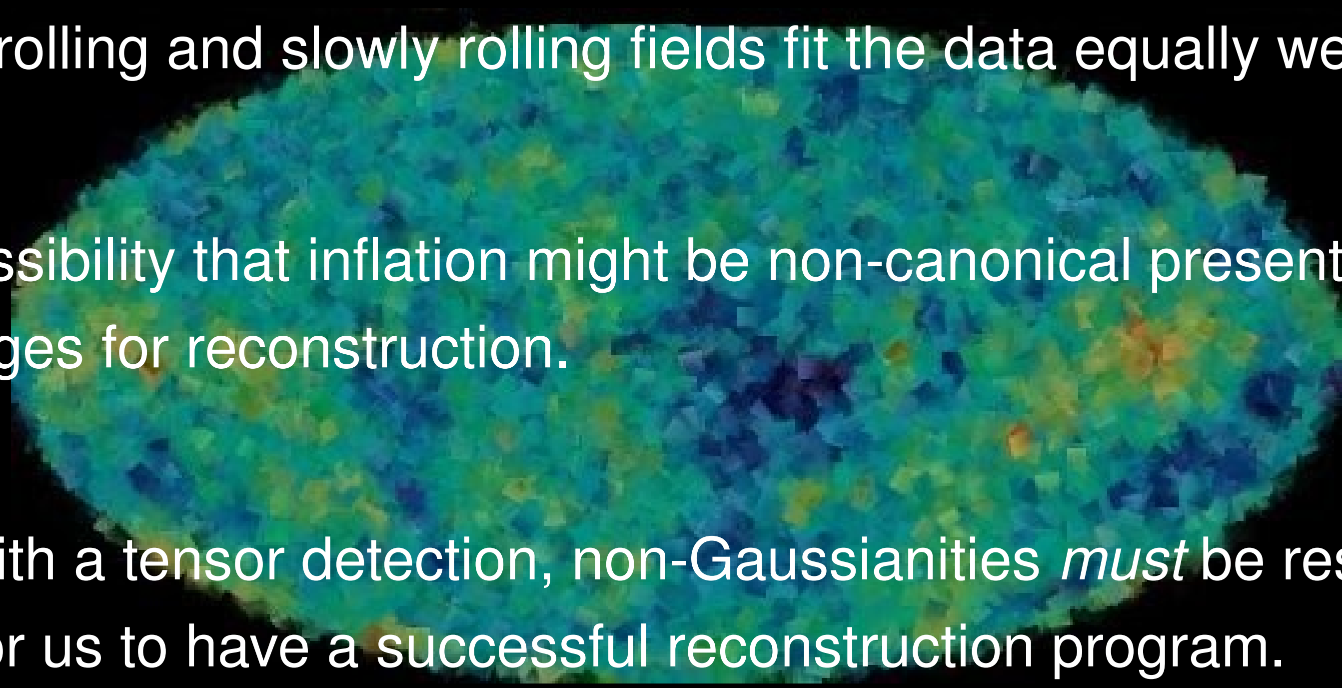
Marginalized Errors

	$V_0 \times 10^9 M_{\rm Pl}^4$	$V_0' \times 10^{10} M_{\rm Pl}^3$	$V_0'' \times 10^{11} M_{\rm Pl}^2$	$-2\ln\mathcal{L}$
canonical	$3.7^{+1.9}_{-0.7}$	$4.3^{+3.7}_{-1.2}$	$2.5^{+7.2}_{-1.8}$	7931.29
non-canonical	$3.6^{+1.9}_{-0.9}$	16^{+10}_{-12}	91^{+80}_{-90}	7931.16

Best-fit Potentials



Conclusions



- Cosmic variance obscures the initial dynamics of the inflaton. Both rapidly rolling and slowly rolling fields fit the data equally well.

- The possibility that inflation might be non-canonical presents new challenges for reconstruction.

- Even with a tensor detection, non-Gaussianities *must* be resolved in order for us to have a successful reconstruction program.

- More guidance from theory will be necessary to make further progress in this case.