A Holographic Dual of Bjorken Flow

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Based on
S. Kinoshita, S. Mukohyama, S.N. and K. Oda,
arXiv:0807.3797

Motivations and background

Motivation

RHIC: Relativistic Heavy Ion Collider
(@ Brookhaven National Laboratory)

Heavy ion:
e.g. ^{197}Au
$\sqrt{S_{NN}}$ ~200GeV.

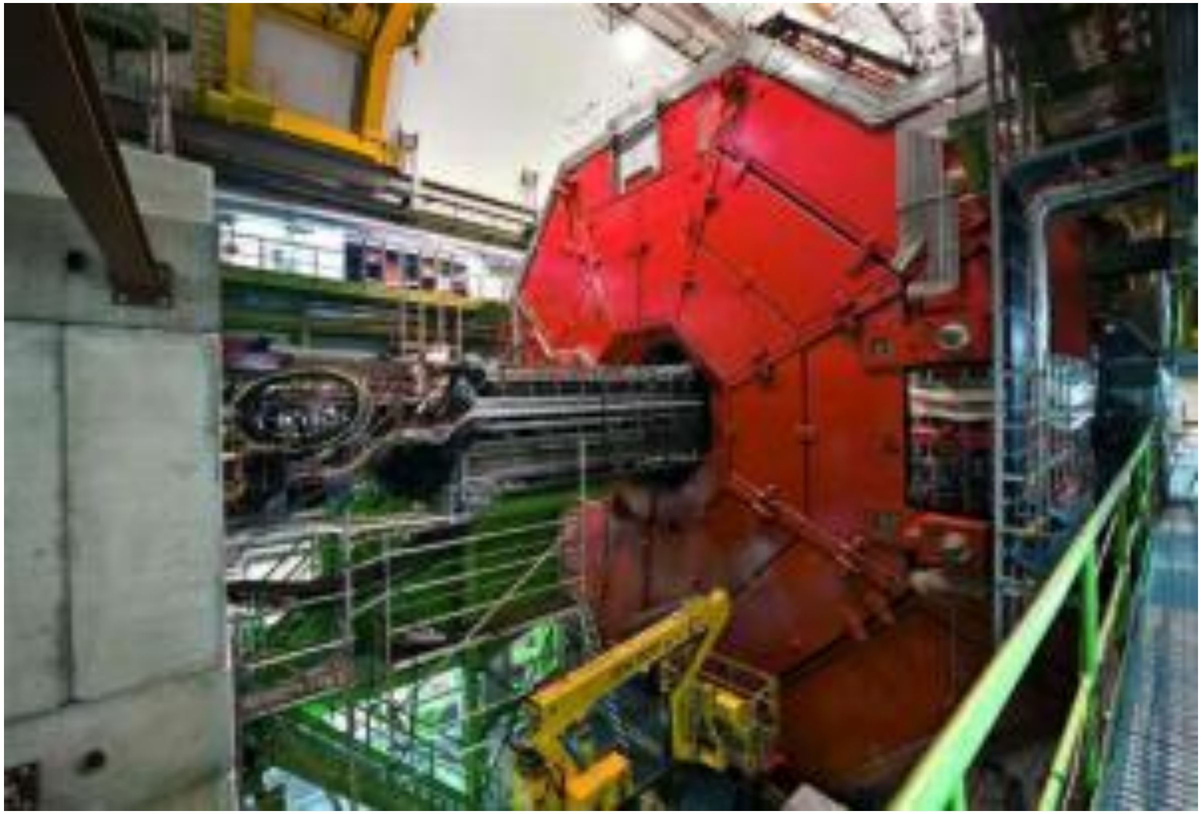
Quark-gluon Plasma (QGP) is observed.

http://www.bnl.gov/RHIC/inside_1.htm

Also at LHC

ALICE

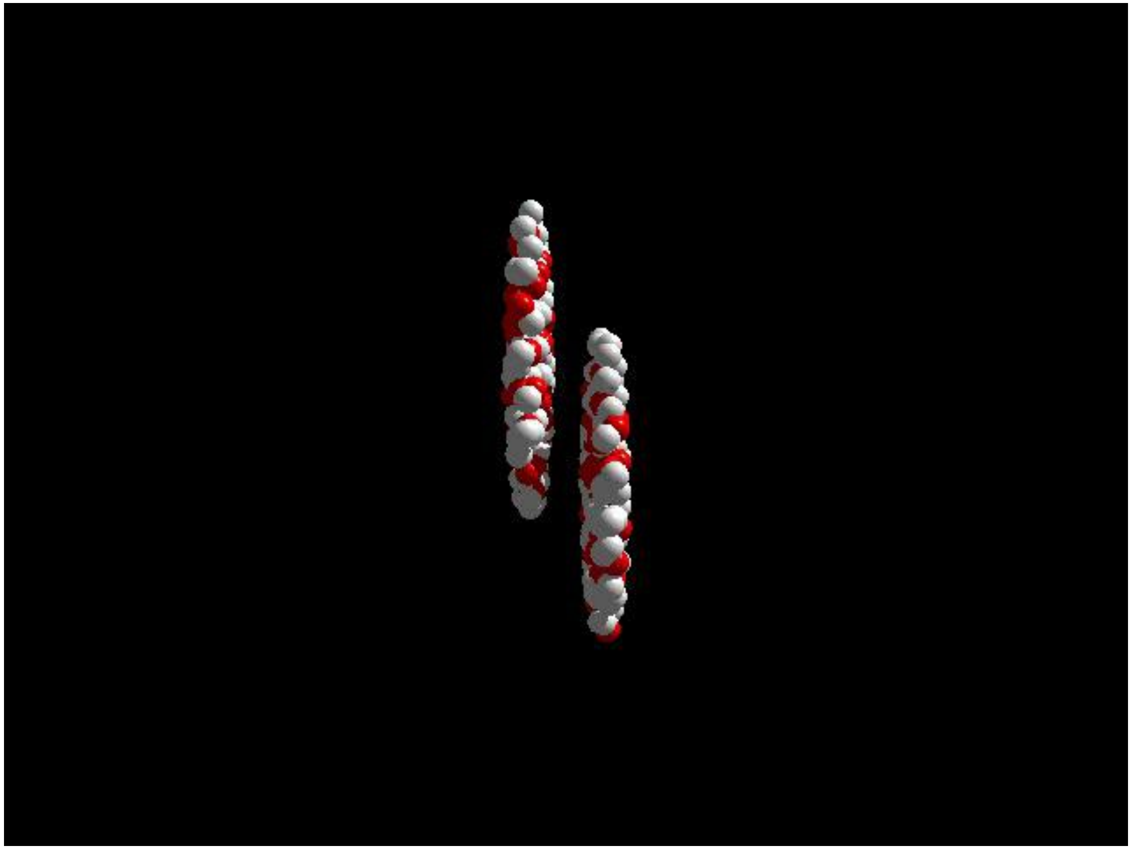
ALICE
ATLAS
CMS



Similar exp. at
- FAIR@ GSI
- NICA@ JINR

http://aliceinfo.cern.ch/Public/en/Chapter4/Chapter4Gallery-en.html

Quark-gluon plasma (QGP) is created as a time-dependent system



http://www.bnl.gov/RHIC/heavy_ion.htm

QGP is a strongly interacting system

BNL press release:

RHIC Scientists Serve Up "Perfect" Liquid

New state of matter more remarkable than predicted
-- raising many new questions
April 18, 2005

Perfect fluidity: zero (very small) viscosity

Shear viscosity → $\frac{\eta}{s} \approx O(0.1)$ Teaney(2003) Hirano-Gyulassy(2005)

The smallest value which has ever been observed.

$$\frac{\eta}{s} = \frac{1}{4\pi} \approx 0.1$$

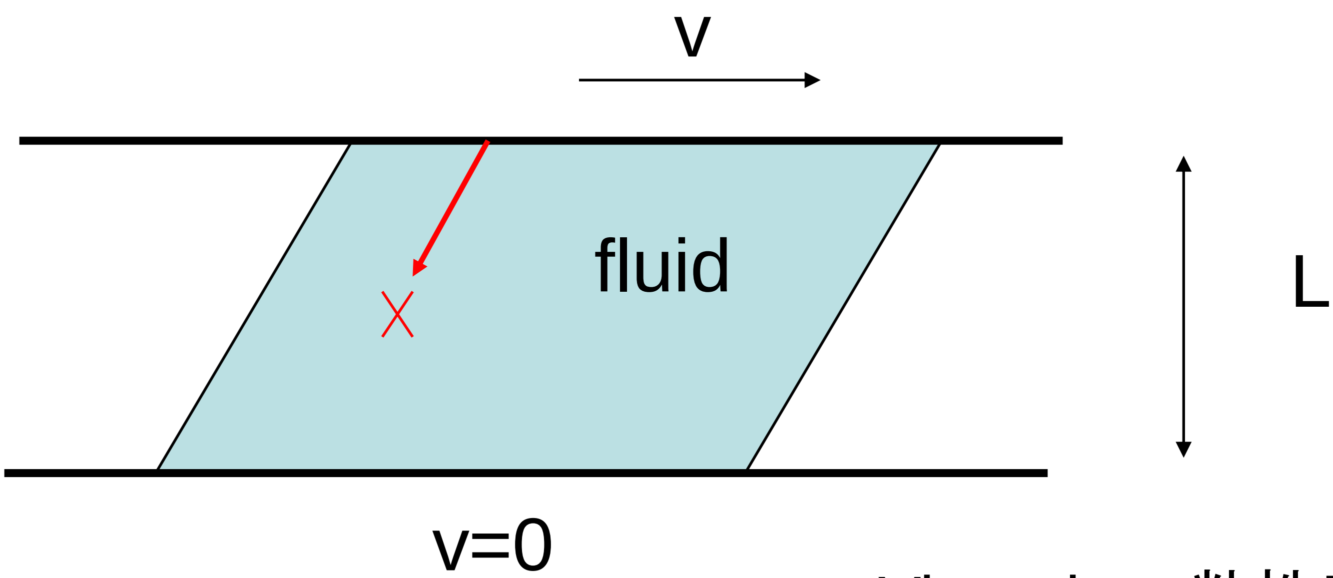
from AdS/CFT

Kovtun-Son-Starinets (2004)

Perturbative QCD says: $\eta \approx \frac{T^3}{g^4 \ln g^{-1}}$

Small viscosity: strong coupling

Shear viscosity



$$\frac{F}{A} = \eta \frac{v}{L}$$

Viscosity: 粘性度

Shear: 剪断

If the interaction is strong:

- the cross section is large,
- the mean free path is short,
- the momentum transfer is suppressed.

Frameworks for strongly interacting YM-theory plasma

- Lattice QCD: a first-principle computation

  However, it is technically difficult to analyze time-dependent systems.

- (Relativistic) Hydrodynamics

  - It works well. However, this is an effective theory for macroscopic physics.

    (entropy, temperature, pressure, energy density,….)

  - Information on microscopic physics is lost.

    (correlation functions of operators, equation of state, transport coefficients such as viscosity,…)

Inputs of hydrodynamics has to be given by other theories.

Alternative framework: AdS/CFT

AdS/CFT

Both macroscopic and microscopic physics of (some) strongly interacting gauge-theory plasma can be described within a single framework of AdS/CFT.

⇩

AdS/CFT may be useful in the research of YM-theory plasma.

However,

Time-dependent AdS/CFT has not yet been established completely.

↓

We deal with N=4 SYM instead of QCD.

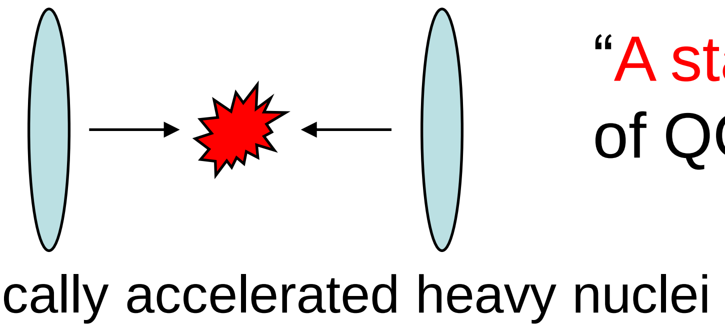
Our work

We construct a time-dependent AdS/CFT that describes the Bjorken flow of plasma of large-Nc, strongly coupled N=4 super Yang-Mills theory.

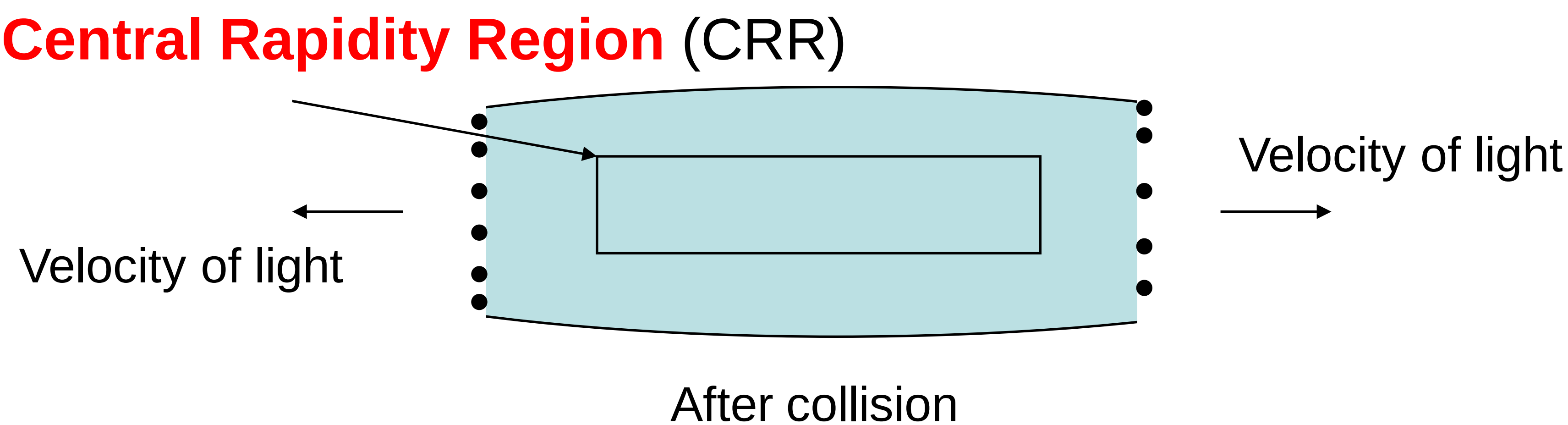
A standard model for the expanding QGP

Bjorken flow:
a simple standard model of the expansion of QGP

Bjorken flow (Bjorken 1983)



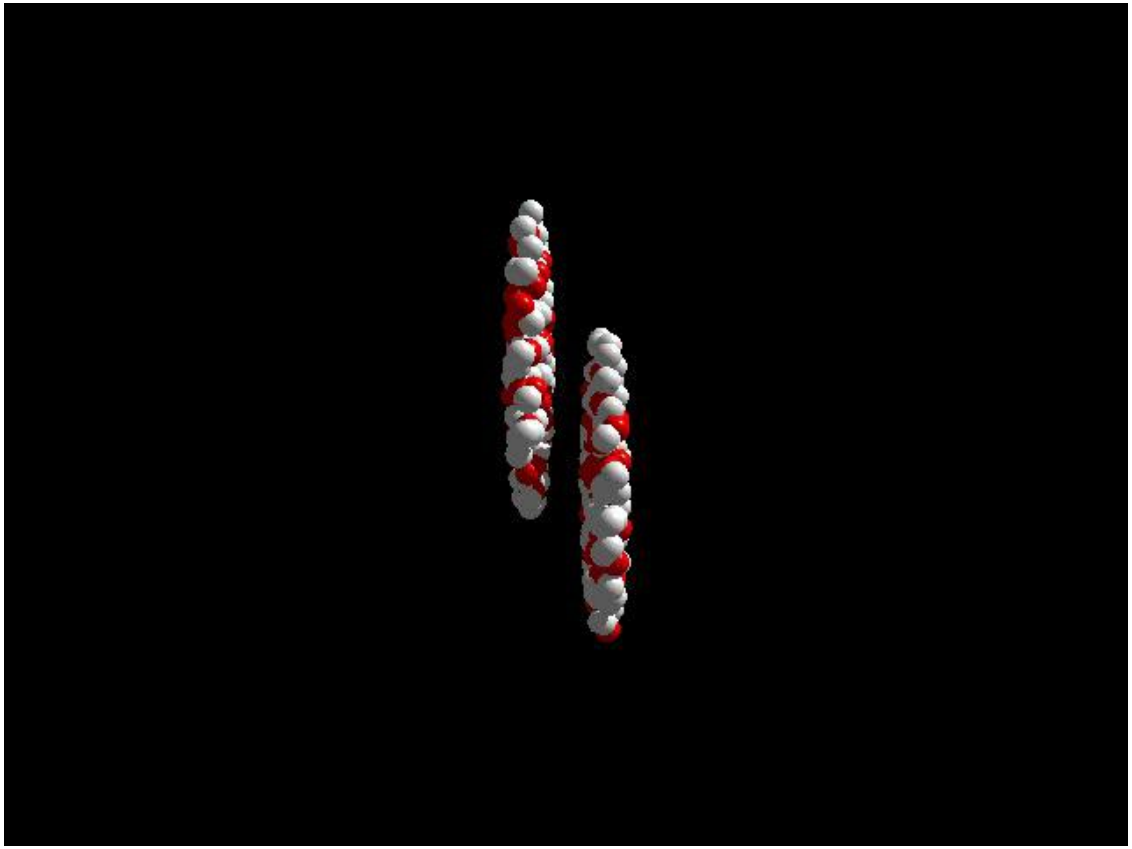
"A standard model" of QGP expansion



- (Almost) **one-dimensional expansion**.
- We have **boost symmetry** in the CRR.

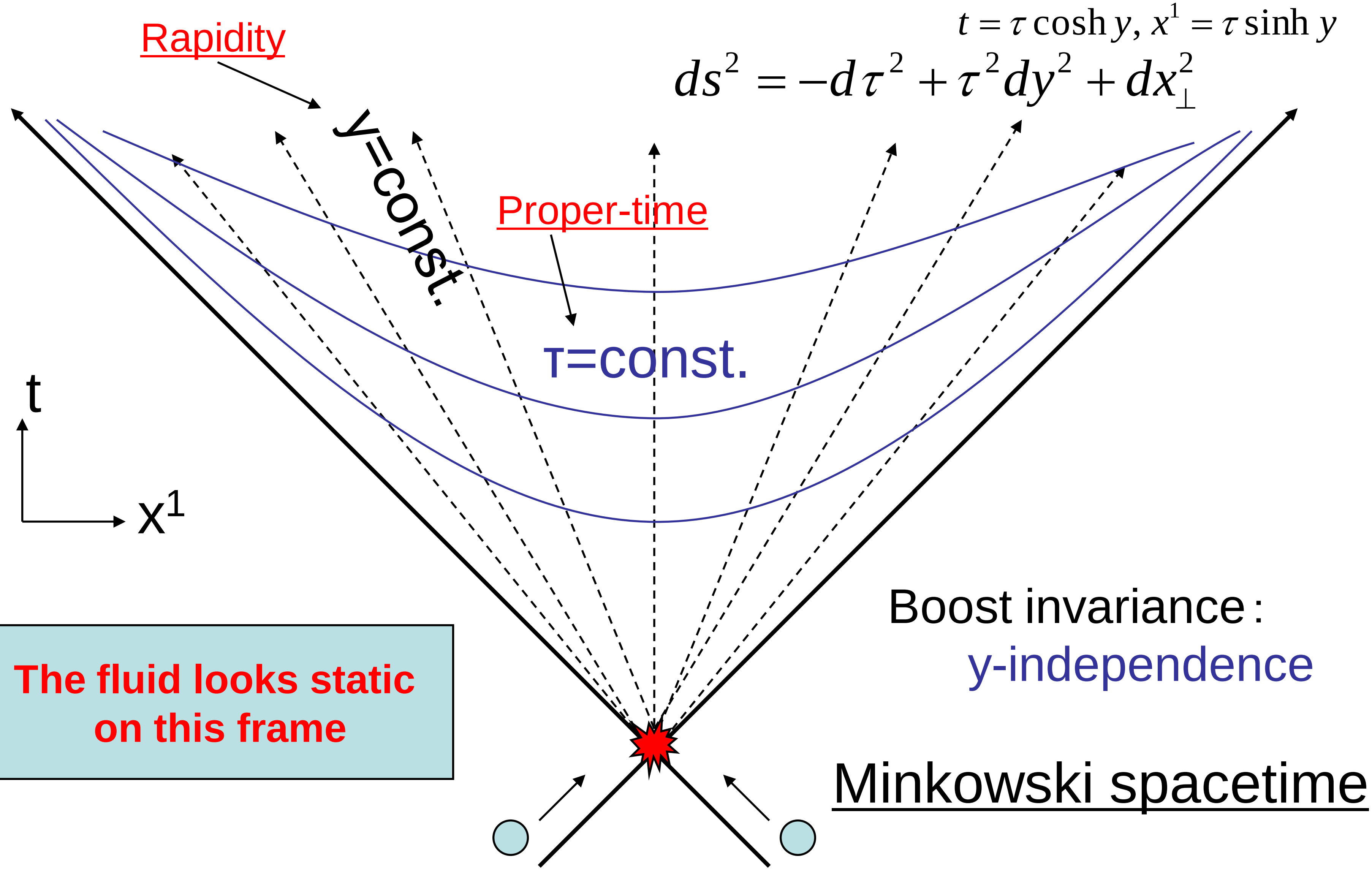
⇨ Time dependence of the physical quantities are written by the **proper time**.

Quark-gluon plasma (QGP) as a one-dimensional expansion



http://www.bnl.gov/RHIC/heavy_ion.htm

Local rest frame(LRF)



The fluid looks static on this frame

Boost invariance : y-independence

Relativistic Hydrodynamics

- We take the local rest frame.

⇨ Our case: (τ, y, x^2, x^3) $t = \tau \cosh y,\ x^1 = \tau \sinh y$

Proper-time (τ), Rapidity (y)

$$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$$

Then, the stress tensor becomes diagonal:

$$T_{\mu\nu} = \begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \tau^2 (p - \frac{4}{3}\frac{\eta}{\tau}) & 0 & 0 \\ 0 & 0 & p + \frac{2}{3}\frac{\eta}{\tau} & 0 \\ 0 & 0 & 0 & p + \frac{2}{3}\frac{\eta}{\tau} \end{pmatrix}$$

energy density (ε), pressure (p), shear viscosity (η)

The bulk viscosity is zero.

3 independent components: $T_{\tau\tau}, T_{yy}, T_{xx}$

2 independent constraints:

$$T^{\mu}_{\mu} = 0$$

"Conformal invariance" (equation of state) $p = \varepsilon / 3$

$$\nabla_{\mu} T^{\mu\tau} = 0$$

Energy-momentum conservation (hydrodynamic equation)

⇩

Only 1 independent component: $T_{\tau\tau}(\tau) \equiv \varepsilon(\tau)$

$$T_{yy} = -\tau^2 \left(T_{\tau\tau} + \tau \partial_{\tau} T_{\tau\tau} \right)$$

$$T_{xx} = \left(T_{\tau\tau} + \frac{1}{2} \tau \partial_{\tau} T_{\tau\tau} \right)$$

To obtain a diff. equation

$$T_{xx} = \varepsilon + \frac{1}{2}\tau\partial_\tau\varepsilon = p + \tfrac{2}{3}\tfrac{\eta}{\tau}$$

$$\varepsilon(\tau) = 3p(\tau)$$

$$\frac{d\varepsilon(\tau)}{d\tau} = -\frac{4}{3}\left(\frac{\varepsilon(\tau)}{\tau} - \frac{\eta(\tau)}{\tau^2}\right)$$

Assumption:

only one quantity (temperature) to determine the scale

$$\varepsilon \propto T^4, \eta \propto T^3$$

$$\eta \equiv \eta_0\varepsilon_0\left(\varepsilon/\varepsilon_0\right)^{3/4}$$

We obtain a differential eq. for ε(τ)

Solution

important

$T \sim \tau^{-1/3}$

$$T_{\tau\tau} \equiv \varepsilon = \varepsilon_0 \left(\frac{1}{\tau^{4/3}} - \frac{2\eta_0}{\tau^2} + \ldots \right)$$

expansion w.r.t

$\tau^{-2/3}$

$$T_{yy} = -\tau^2 \left(T_{\tau\tau} + \tau \partial_\tau T_{\tau\tau} \right)$$

$$T_{xx} = \left(T_{\tau\tau} + \frac{1}{2} \tau \partial_\tau T_{\tau\tau} \right)$$

Once the parameters (transport coefficients) are given, $T_{\mu\nu}(\tau)$ is completely determined.

But, hydro cannot determine them.

Towards time-dependent AdS/CFT

(Anti de Sitter)

AdS/CFT

(Weak version)

(Conformal Field Theory)

Classical Supergravity on $AdS_5 \times S^5$

|| conjecture

Maldacena (1997)

4dim. Large-Nc SU(Nc)

N=4 Super Yang-Mills

at the large 't Hooft coupling

Strongly interacting quantum YM !!

AdS/CFT at finite temperature

Classical Supergravity on

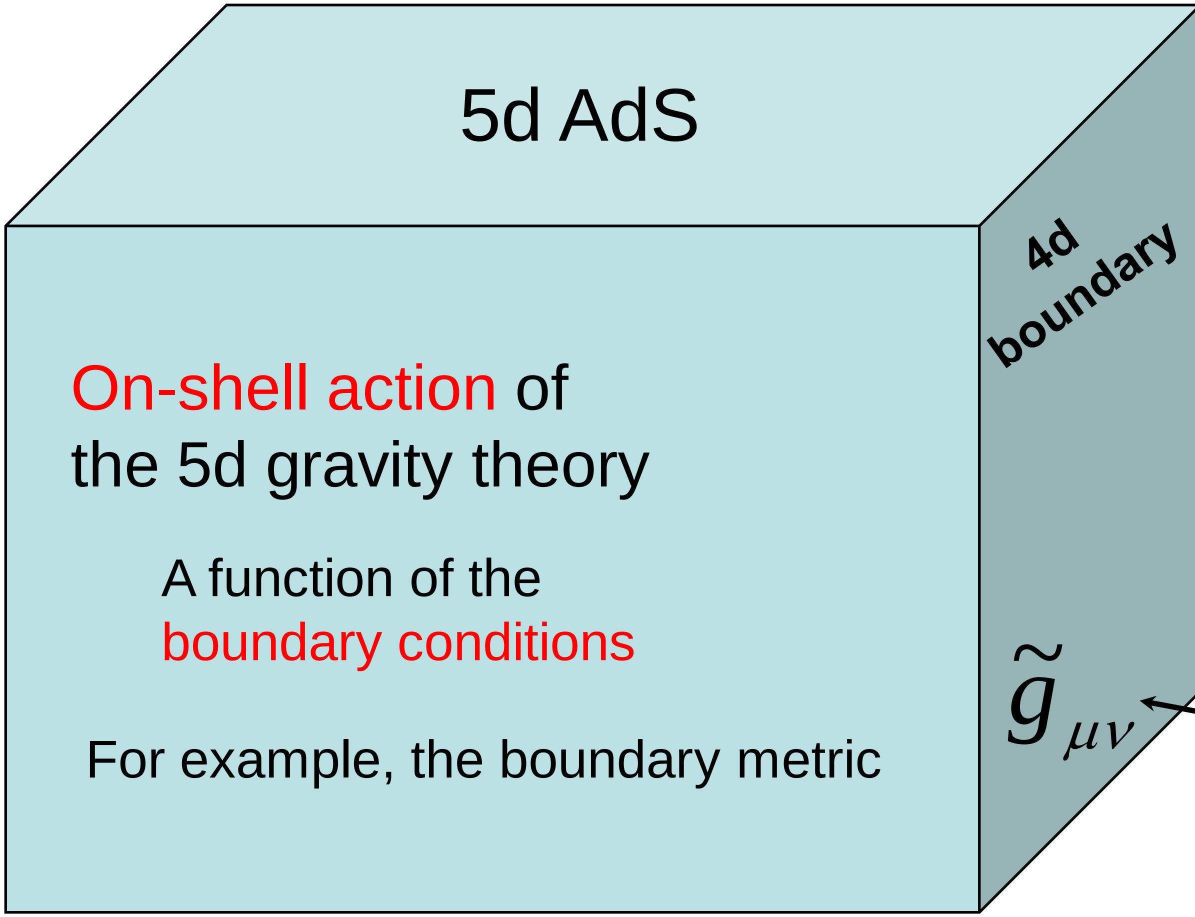
AdS black hole $\times S^5$

|| conjecture

Witten (1998)

4dim. Large-Nc strongly coupled SU(N_c) N=4 SYM at finite temperature (in the deconfinement phase).

AdS/CFT dictionary



Effective action of the 4d gauge theory

A function of sources

Expectation value of an operator by differentiating w.r.t. the source.

$$\langle T^{\mu\nu} \rangle$$

4d stress tensor

The stress tensor of the plasma is given by the geometry.

Hydrodynamics

Hydrodynamics describes dynamics of conserved currents such as stress tensor.

⇩

We know that the 4d stress tensor can be read from the dual geometry.

Hydrodynamics may be given by
the dynamics of the dual geometry.

Towards time-dependent AdS/CFT

How to obtain the geometry?

The bulk geometry is obtained by solving the equations of motion of 5d Einstein gravity with $\Lambda<0$ with appropriate boundary data.

Bjorken's case:

- The boundary metric is that of the comoving frame: $$ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$$

- The 4d stress tensor is diagonal on this frame.

We set (the 4d part of) the bulk metric diagonal. (ansatz)

This tells our fluid undergoes the Bjorken flow.

Time-dependent AdS/CFT

Earlier works

A time-dependent AdS/CFT

A time-dependent geometry that describes Bjorken flow of N=4 SYM fluid was first obtained within a late-time approximation by Janik-Peschanski.

Janik-Peschanski, hep-th/0512162

They have used Fefferman-Graham coordinates:

$$ds^2 = \frac{\tilde{g}_{\mu\nu}(\tau, z)dx^\mu dx^\nu + dz^2}{z^2}$$

geometry as a solution

$$\tilde{g}_{\mu\nu}(\tau, z) = \tilde{g}^{(0)}_{\mu\nu}(\tau) + \tilde{g}^{(4)}_{\mu\nu}(\tau) z^4 + \dots\dots\dots\dots$$

4d geometry (LRF)

stress tensor of YM

boundary condition to 5d Einstein's equation with $\Lambda<0$

Unfortunately, we cannot solve exactly

They emploed the late-time approximation:

$$\tau \to \infty, \text{ with } \frac{z}{\tau^{p/4}} \equiv v \text{ fixed}$$

Janik-Peschanski
hep-th/0512162

$\tilde{g}_\tau^{\ \tau}, \tilde{g}_y^{\ y}, \tilde{g}_x^{\ x}$ have the structure of

$$f^{(1)}(v) + f^{(2)}(v)\tau^{(p-4)/4} + \ldots\ldots\ldots$$

We discard the higher-order terms.

Janik-Peschanski's result

$$ds^2 = \frac{1}{z^2}\left[-\frac{(1-\frac{\varepsilon z^4}{3})^2}{1+\frac{\varepsilon z^4}{3}} d\tau^2 + (1+\frac{\varepsilon z^4}{3})\left(\tau^2 dy^2 + d\vec{x}_\perp^2\right) \right] + \frac{dz^2}{z^2}$$

5th coordinate

$$\varepsilon(\tau) = \varepsilon_0 \tau^{-4/3} + \ldots$$ ⇦ from relativistic hydrodynamics

The statement

If we start with unphysical assumption like

$$\varepsilon(\tau) = \varepsilon_0 \tau^{-p}, \text{ with } p \neq 4/3,$$

the obtained geometry is singular:

$$R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda} = \infty$$ at the point $g_{\tau\tau}$=0. (horizon?)

Regularity of the spacetime determines the correct parameters.

Many success

For example:

- 1st order: Introduction of the shear viscosity:

S.N. and S-J.Sin, hep-th/0607123

- 2nd order: Determination of $\frac{\eta}{S} = \frac{1}{4\pi}$ from the regularity:

Janik, hep-th/0610144

same as KSS

- 3rd order: Determination of the relaxation time from the absence of the power singularity:

Heller and Janik, hep-th/0703243

But, a serious problems came out.

- An un-removable logarithmic singularity appears at the third order.

  (Benincasa-Buchel-heller-Janik, arXiv:0712.2025)

This suggests that the late-time expansion they are using is not consistent.

We have many questions, too.

- Is the time-dependent geometry really a black hole?

- Why do we need the regul

String correction:
e.g. $l_s^{\,4} R_{\mu\nu\rho\lambda} R^{\mu\nu\rho\lambda}$

From the viewpoint of the or
the singularity merely means a breakdown of the super-gravity approximation.

No a priori reason to forbid the physics because of the singularity.

We will try to answer these questions and resolve the problem of the logarithmic divergence.

Our work

What is black hole?

Definition: presence of an event horizon.

(Future) Event horizon:

No signals from inside the (future) event horizon can reach the observer at the future infinity.

We need to examine the global structure of the spacetime to determine the location of the horizon.

It is not easy to examine the presence of event horizon especially in time-dependent systems, in general.

Another horizon

Apparent horizon:

The boundary surface of a region in which the light ray directed outward is moving inward.

trapped region

- This is defined locally.
- But, this is a coordinate-dependent concept.
- This can be different from the event horizon in time-dependent systems.

Some comments

- Usually, if we have an apparent horizon, an event horizon is located outside, or on top of it.

(Ref. Hawking-Ellis)

The presence of an apparent horizon is a sufficient condition for the presence of an event horizon.

Let us examine the apparent horizon of the dual geometry.

Location of the apparent horizon

Normalization:
we may omit this factor
in this talk.

the apparent horizon is given by

$$e^{F}\vartheta_{+}\vartheta_{-}=0,$$

"expansion"

$$\theta_{\pm}\propto\left(g^{\tau\tau}\partial_{\tau}\pm g^{zz}\partial_{z}\right)\log\left(\sqrt{g_{yy}g_{\perp\perp}g_{\perp\perp}}\right),$$

derivative along the null direction

volume element of the 3d surface

$\vartheta_{+}\vartheta_{-}<0$: un-trapped region

$\vartheta_{+}\vartheta_{-}>0$: trapped region

However, for Janik-Peschanski's Geometry

$$e^F \theta_+ \theta_- = -\frac{9}{2}\left(\frac{3 - z^4/\tau^{4/3}}{3 + z^4/\tau^{4/3}}\right)^2 \leq 0 \qquad \text{at } p = 4/3$$

Always in the un-trapped region!

No trapped region !

What is going on?

Janik-Peschanski (and all other related works) are based on Fefferman-Graham coordinates.

$$ds^2 = \frac{-Ad\tau^2 + Bdy^2 + Cdx_\perp{}^2 + dz^2}{z^2}$$

Static AdS-BH

Fefferman-Graham coordinates

$$ds^2 = \frac{1}{z^2}\left[-\frac{(1-\frac{z^4}{z_0^4})^2}{1+\frac{z^4}{z_0^4}}\,d\tau^2 + (1+\frac{z^4}{z_0^4})\left(\tau^2 dy^2 + dx_\perp^2\right)\right] + \frac{dz^2}{z^2}$$

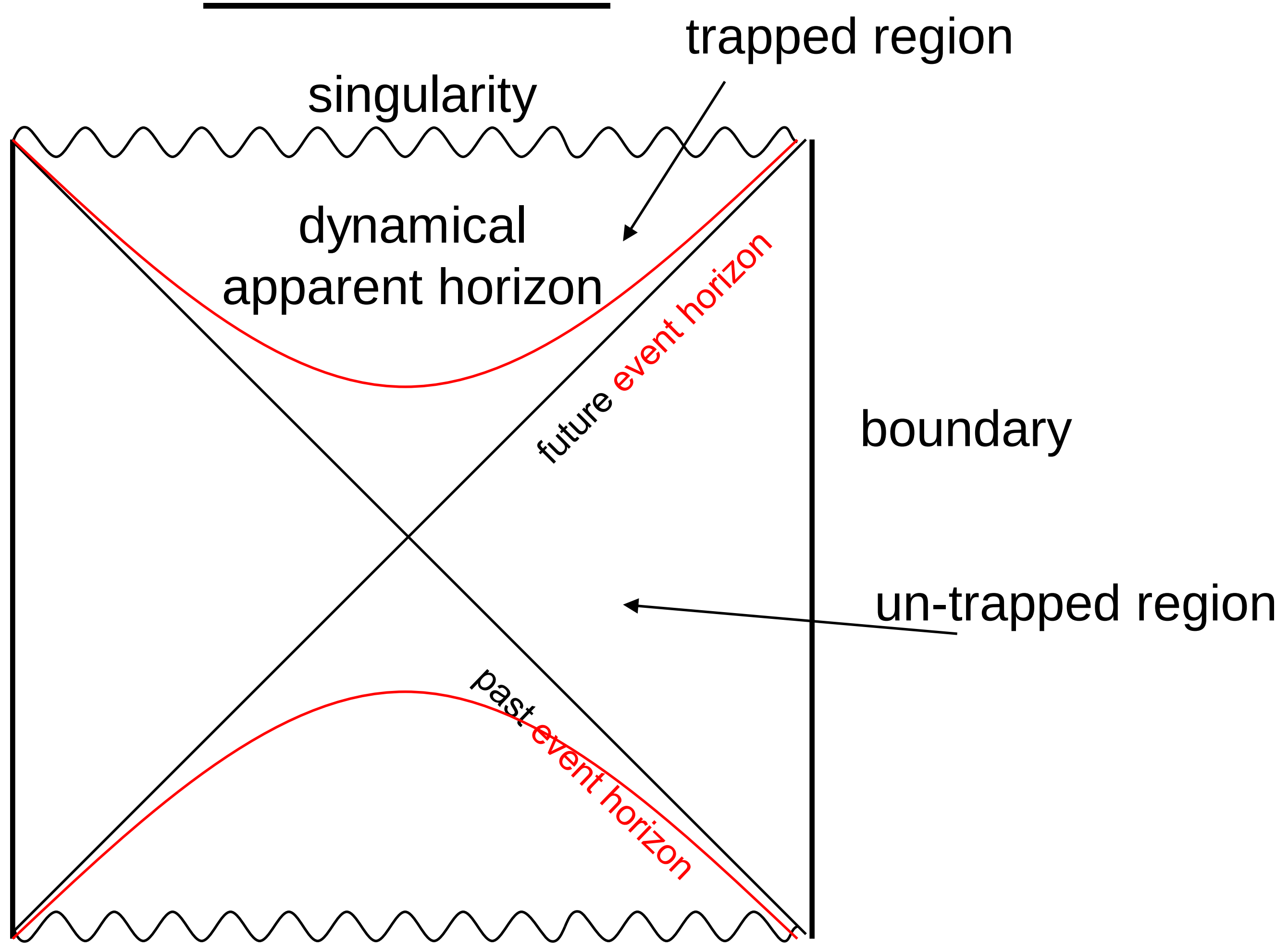
Only outside the horizon!

$$r = \frac{\sqrt{z_0^2/z^2 + z^2/z_0^2}}{z_0} \geq \frac{\sqrt{2}}{z_0} = r_0$$

Schwarzschild coordinates

$$ds^2 = -r^2\left(1-\frac{r_0^4}{r^4}\right)d\tau^2 + r^2\left(\tau^2 dy^2 + dx_\perp^2\right) + r^{-2}\left(1-\frac{r_0^4}{r^4}\right)^{-1} dr^2$$

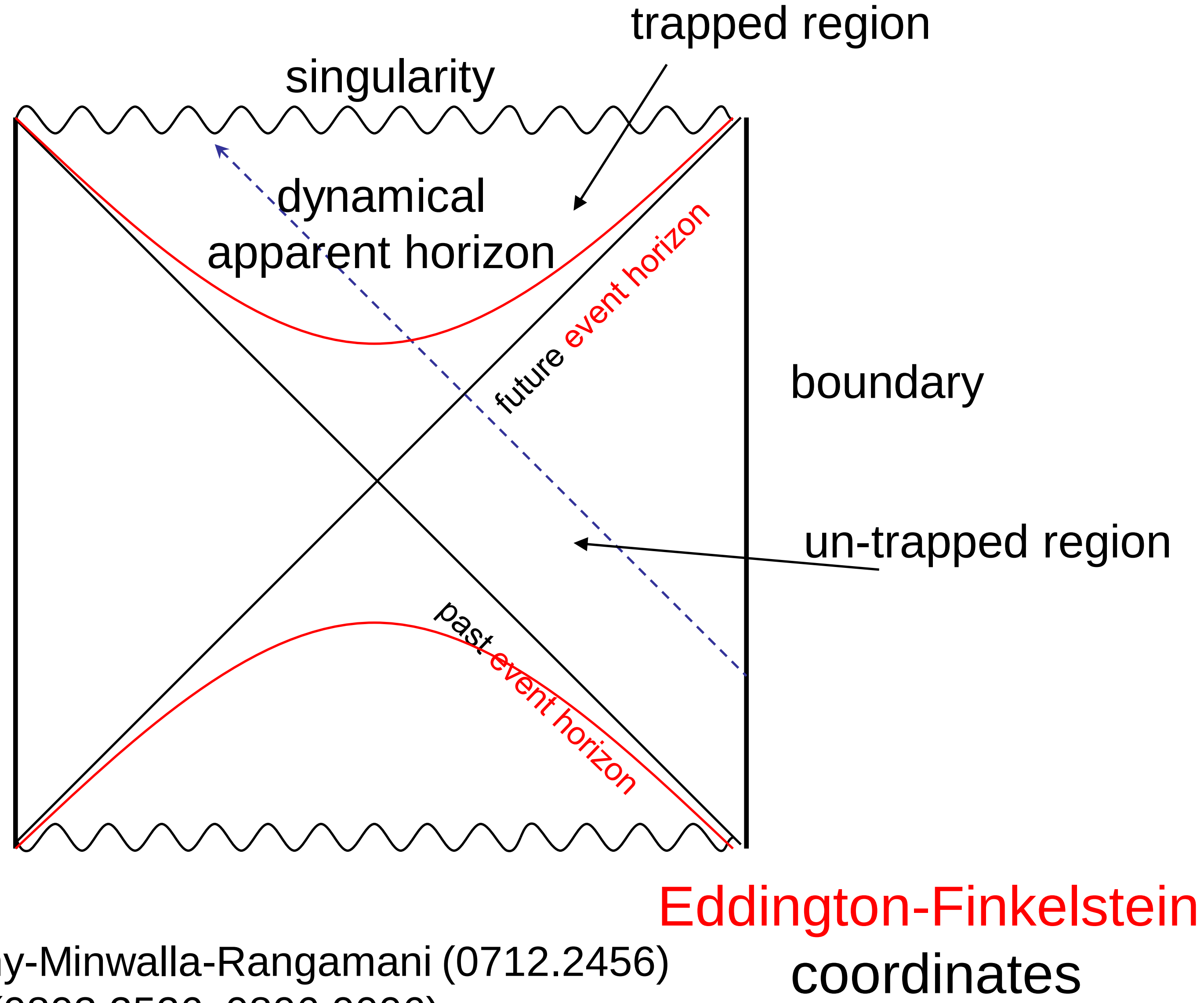
AdS-BH


trapped region
singularity
dynamical
apparent horizon
future event horizon
boundary
un-trapped region
past event horizon

What we have found so far:

- The Fefferman-Graham coordinates are not a good coordinate system for analysis of horizons (no trapped region included).
- The geometry inside the horizon cannot be obtained from the Janik's results, anyway.
- We need a better coordinate system.

Better coordinates?



Cf.
Bhattacharyya-Hubeny-Minwalla-Rangamani (0712.2456)
Bhattacharyya et. al. (0803.2526, 0806.0006)

Eddington-Finkelstein coordinates

Static AdS-BH:

$$ds^2 = -r^2\left(1 - \frac{r_0^4}{r^4}\right)dt^2 + 2dtdr + r^2 d\vec{x}^2$$

At least for the static case,

- There is no coordinate singularity.
- The trapped region and the un-trapped region are on the same coordinate patch.

  (We can safely analyze the location of the horizon.)

Our proposal

Parametrization of the dual geometry:

$$ds^2 = -r^2 \tilde{g}_{\tau\tau}\, d\tau^2 + 2d\tau\, dr + r^2 \tilde{g}_{yy} dy^2 + r^2 \tilde{g}_{xx} d\vec{x}_\perp^2$$

We assume they depend only on τ, because of the symmetry.

The 5d Einstein's eq. gives differential equations of $\tilde{g}_{\mu\nu}$.

We solve them under the following boundary condition:

$\tilde{g}_{\tau\tau} \to 1, \quad \tilde{g}_{yy} \to \tau^2, \quad \tilde{g}_{xx} \to 1,$ at the boudary (r= ∞).

⇕

boundary metric: $ds^2 = -d\tau^2 + \tau^2 dy^2 + dx_\perp^2$

Asymptotic solution around the boundary

$$-\tilde{g}_{\tau\tau} = 1 + h\,r^{-1} + \left(h^2\!/\!_4 - \partial_\tau h\right) r^{-2} + f\, r^{-4} + \ldots.$$

$$\tilde{g}_{yy} = \tau^2 + \tau\left(\tau h + 2\right) r^{-1} + \tfrac{1}{4}\left(\tau h + 2\right)^2 r^{-2} + \tau^2\left(f + \tfrac{3}{4}\tau\,\partial_\tau f\right) r^{-4} + \ldots$$

$$\tilde{g}_{xx} = 1 + h\,r^{-1} + \frac{h^2}{4} r^{-2} - \tfrac{1}{2}\left(f + \tfrac{3}{4}\tau\,\partial_\tau f\right) r^{-4} + \ldots$$

where h and f are functions of τ.

Only two functions f(τ) and h(τ) so far….

The stress tensor

The 4d ... ns of

the bou...

$$T_{\mu\nu} = r^2\left(\tfrac{N_c^2}{4\pi^2}\right)\left(K_{\mu\nu} - K\gamma_{\mu\nu} - 3\gamma_{\mu\nu} + \tfrac{1}{2}G_{\mu\nu}\right)\Big|_{r\to\infty}$$

...2121.)

For our case:

$$T_{\tau\tau} = -\varepsilon_0 f(\tau)$$

$$T_{yy} = \varepsilon_0 \tau^2\left(f(\tau) + \tau\partial_\tau f(\tau)\right)$$

$$T_{xx} = -\varepsilon_0\left(f(\tau) + \frac{1}{2}\tau\partial_\tau f(\tau)\right)$$

consistent with hydro eq.

How about h?

h(τ) turns out to be a gauge degree of freedom.

$r \to r + \xi(\tau)$ does not modify the structure of

$$ds^2 = -r^2\tilde{g}_{\tau\tau}\,d\tau^2 + 2d\tau\,dr + r^2\tilde{g}_{yy}dy^2 + r^2\tilde{g}_{xx}d\vec{x}_\perp^2$$

The hydrodynamic equation and the equation of state in the gravity dual

The 5d Einstein's equation around the boundary yields the hydrodynamic equation (and the equation of state).

See also, Bhattacharyya-Hubeny-Minwalla-Rangamani (0712.2456)

Einstein's eq. $\Longrightarrow$ $\nabla_\mu T^{\mu\nu} = 0$

How much does the gravity know the fluid dynamics of the YM plasma?

Then, how to obtain the transport coefficients?

Can we determine f(τ) as a function of τ?

(Yes.)

- We need a global (analytic in r) solution.
- A better parametrization:

$$ds^2 = -r^2 a\, d\tau^2 + 2d\tau\, dr + r^2 \tau^2 e^{2b-2c}\left(1+(r\tau)^{-1}\right)^2 dy^2 + r^2 e^c d\vec{x}_\perp^2$$

The boundary condition: $a \to 1, \quad b \to 0, \quad c \to 0,$ at r= ∞.

Let us solve the Einstein's equation.

Late-time approximation

It is very difficult to obtain the solution analytic in τ.

We introduce a late-time approximation by making an analogy with what Janik-Peschanski did on the FG coordinates.

Janik-Peschanski:

$\tau^{-2/3}$ expansion with $z\tau^{-1/3} = v$ fixed.

Now, $r \sim z^{-1}$.

Let us employ $\tau^{-2/3}$ expansion with $r\tau^{1/3} = u$ fixed.

Our late-time approximation

More explicitly,

$$ds^2 = -r^2 a\, d\tau^2 + 2d\tau\, dr + r^2 \tau^2 e^{2b-2c}\left(1+(r\tau)^{-1}\right)^2 dy^2 + r^2 e^c d\vec{x}_\perp^2$$

We solve the differential equations for a(τ,u), b(τ,u), c(τ,u) order by order:

$$a(\tau,u) = a_0(u) + a_1(u)\tau^{-2/3} + a_2(u)\tau^{-4/3} + \ldots\ldots$$

zeroth order   first order   second order

(similar for b and c)

($u = r\tau^{1/3}$)

The zeroth-order solution

$$w^4 = \varepsilon_0 \left(\frac{3N_c^2}{8\pi^2} \right)$$

$$ds^2 = -r^2\left(1 - \frac{w^4}{(r\tau^{1/3})^4}\right)d\tau^2 + 2d\tau dr + r^2(\tau^{2/3}dy^2 + dx_\perp^2)$$

This is u

- This reproduces the correct zeroth-order stress tensor of the Bjorken flow.

$$T_{\tau\tau} = \varepsilon = \varepsilon_0\left(\frac{1}{\tau^{4/3}} + \ldots\right)$$

- We have an apparent horizon.

$$e^F \theta_+ \theta_- = -\frac{9}{2}\left(1 - u^{-4}w^4\right)$$

trapped region if u<w.

The location of the apparent horizon: $u = w + O(\tau^{-2/3})$

The (event) horizon is necessary

$$\left(R_{\mu\nu\rho\lambda}\right)^2 = 8\left(5 + \frac{9w^8}{u^8}\right) + O(\tau^{-2/3})$$

We have a physical singularity at the origin.

However, this is hidden by the apparent horizon at u=w hence the event horizon (outside it).

⇨ Not a naked singularity.

OK, from the viewpoint of the cosmic censorship hypothesis.

The first-order solution

$$ds^2 = -r^2 a\, d\tau^2 + 2d\tau\, dr + r^2\tau^2 e^{2b-2c}\left(1+(r\tau)^{-1}\right)^2 dy^2 + r^2 e^c d\vec{x}_\perp^2$$

$$a_1 = -\frac{2}{3}\frac{(1+\xi_1)u^4 + \xi_1 w^4 - 3\eta_0 u w^4}{u^5}$$

$$b_1 = -\frac{1+\xi_1}{u}$$

gauge degree of freedom

$$c_1 = \frac{1}{3w}\left\{\arctan(uw^{-1}) - \pi/2 + 1/2\log(u-w) - 1/2\log(u+w)\right\}$$
$$- \eta_0/2\,\log(1-w^4u^{-4}) - 2\xi_1/3u$$

c$_1$ is regular at u=w, only when $\eta_0 = \frac{1}{3w}$.

Regularity of c_1 is necessary.

We can show

$$R^{y}{}_{\mu y \nu} N^\mu N^\nu = \frac{c_1'}{u} + \frac{1}{2} c_1'' + \text{regular} + O(\tau^{-2/3})$$

$$= -\frac{1-3w\eta_0}{12w(u-w)^2} + \frac{1-3w\eta_0}{6w^2(u-w)} + \text{regular} + O(\tau^{-2/3})$$

$$N^\mu = -\frac{1}{\sqrt{2}}\left(1,0,0,0,\frac{r^2 a+2}{2}\right)$$: a regular space-like unit vector

Riemann tensor projected onto a regular orthonormal basis

This has to be regular to make the geometry regular.

What is this value?

Gubser-Klebanov-Peet, hep-th/9602135

$$\varepsilon = \frac{3}{8}\pi^2 N_c^2 T^4 \longrightarrow s = \frac{1}{2}\pi^2 N_c^2 T^3$$

First law of thermodynamics

Our definition and result:

$$\eta = \eta_0 \varepsilon_0 \left(\varepsilon / \varepsilon_0 \right)^{3/4} \qquad w^4 = \varepsilon_0 \left(\frac{3N_c^2}{8\pi^2} \right)$$

Combine all of them:

$$\frac{\eta}{s} = \frac{1}{4\pi} 3\eta_0 w \xrightarrow{\eta_0 = \frac{1}{3w}} \boxed{\frac{\eta}{s} = \frac{1}{4\pi}}$$

The famous ratio by Kovtun-Son-Starinets (2004)

Second-order results:

- We have obtained the solution explicitly, but it is too much complicated to exhibit here.

- From the regularity of the geometry, "relaxation time" is uniquely determined.

  consistent with Heller-Janik, Baier et. al., and Bhattacharyya et. al.

  2nd-order transport coefficient

- $$\left(R_{\mu\nu\rho\lambda}\right)^2 = \frac{4(9\eta_0^2 w^2 - 1)}{3(u-w)}\left(\frac{1}{u-w} - 2\right)\tau^{-4/3} + \mathrm{regular} + O(\tau^{-2})$$

$$
\begin{aligned}
a_2(u) = {} & \frac{\left(u^4-3w^4\right)\xi_1^2}{9u^6} - \frac{4\left(u^3-3w^4\eta_0\right)\xi_1}{9u^5} - \frac{2\left(u^4+w^4\right)\xi_2}{3u^5} \\
& - \frac{\left(u^4-2w^3u+w^4\right)\left(9w^2\eta_0^2-1\right)}{12u^5w}\log(u-w) \\
& + \frac{\left(u^4+2w^3u+w^4\right)\left(9w^2\eta_0^2-1\right)}{12u^5w}\log(u+w) \\
& + \frac{\left(u^4+w^4\right)\left(9w^2\eta_0^2+1\right)}{6u^5w}\arctan\left(\frac{u}{w}\right) \\
& + \frac{9\eta_0^2w^4+w^2}{6u^4}\log\left(u^2+w^2\right) \\
& - \frac{3\eta_0\left(3u(12\log u+5)\eta_0+4\right)w^4+4\left(3u\lambda w^4+u^3\right)}{18u^5},
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
b_2(u) = {} & \frac{1}{2u^2} - \frac{\xi_1^2}{6u^2} - \frac{\xi_2}{u} + \frac{\eta_0}{4}\left(-24\eta_0\log u - \frac{4}{u} + \frac{\pi}{w}\right) \\
& + \frac{\left(3w\eta_0-1\right)\left(2u-3w+3(4u-3w)w\eta_0\right)}{24uw^2}\log(u-w) \\
& + \frac{\left(3w\eta_0+1\right)\left(-2u-3w+3w(4u+3w)\eta_0\right)}{24uw^2}\log(u+w) \\
& + \frac{1}{12}\left(18\eta_0^2+\frac{1}{w^2}\right)\log\left(u^2+w^2\right) + \frac{9w^2\eta_0^2-2u\eta_0+1}{4uw}\arctan\left(\frac{u}{w}\right),
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
c_2'(u) = {} & \frac{\left(6\left(w^4-5u^4\right)\eta_0w^4+4u^3\left(u^4+w^4\right)\right)\xi_1}{9\left(u^5-uw^4\right)^2} + \frac{2\xi_1^2}{9u^3} + \frac{2\xi_2}{3u^2} \\
& + \frac{\eta_0\left(12w\eta_0u^5-6wu^4+\pi\left(u^4-w^4\right)u+2w^5\right)w^3}{3\left(u^5-uw^4\right)^2} \\
& + \frac{4\eta_0u^2\log u}{3\left(u^4-w^4\right)} - \frac{3\eta_0u^3+w^2}{9u^5-9uw^4}\log\left(u^2+w^2\right) - \frac{\pi u^3-3w\left(4\lambda w^4+u^2\right)}{9\left(u^5-uw^4\right)w} \\
& - \frac{\left(3w\eta_0-1\right)\left(\left(u+w\right)\left(u^2-2wu+3w^2\right)-9(u-w)w\left(u^2+w^2\right)\eta_0\right)}{36u^2(u-w)\left(u^2+w^2\right)w}\log(u-w) \\
& - \frac{\left(3w\eta_0+1\right)\left(\left(u-w\right)\left(u^2+2wu+3w^2\right)+9w(u+w)\left(u^2+w^2\right)\eta_0\right)}{36u^2(u+w)\left(u^2+w^2\right)w}\log(u+w) \\
& + \frac{u^4+3w^4-3w^2\eta_0\left(4uw^2+9\left(u^4-w^4\right)\eta_0\right)}{18u^2\left(u^4-w^4\right)w}\arctan\left(\frac{u}{w}\right).
\end{aligned} \tag{3}
$$

Second-order results:

- We have obtained the solution explicitly, but it is too much complicated to exhibit here.

- From the regularity of the geometry, "relaxation time" is uniquely determined.

consistent with Heller-Janik, Baier et. al., and Bhattacharyya et. al.

2nd-order transport coefficient

- $$\left(R_{\mu\nu\rho\lambda}\right)^2 = \frac{4(9\eta_0^2 w^2 - 1)}{3(u-w)}\left(\frac{1}{u-w} - 2\right)\tau^{-4/3} + \text{regular} + O(\tau^{-2})$$

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All-order results:

We have shown by using induction that:

- We can always choose (the combination of) the n-th order transport coefficients in such a way that the dual geometry is regular at u=w to the n-th order.
- The geometry becomes singular at u=w if we take other value of (the combination of) the n-th order transport coefficients.
- This means that there is no logarithmic singularity which meant the inconsistency of the analysis on the FG coordinates.

Our model is totally consistent and healthy!

How the induction works

n-th order Einstein's equation:

Diff eq. for n-th order metric
= source which contains only
k(<n)-th order metric

We can always choose an integration constant
(that corresponds to a transport coeff.)
to make c_n regular.

All-order results:

We have shown by using induction that:

- We can always choose (the combination of) the n-th order transport coefficients in such a way that the dual geometry is regular except at the origin.

- The geometry becomes singular at u=w if we take other value of (the combination of) the n-th order transport coefficients.

- This means that there is no logarithmic singularity which meant the inconsistency of the analysis on the FG coordinates.

Our model is totally consistent and healthy!

Coordinate transformation cannot change physics

- Regular coordinate transformation cannot change physics.

- However, the coordinate transformation from Janik's coordinates (FG coordinates) to ours (EF coordinates) is singular at the horizon.

Area of the apparent horizon

$$A_{ap} = w^3\left[1 - \frac{1}{2w}\tau^{-2/3} + \frac{1}{24w^2}\left(2+\pi+6\log 2\right)\tau^{-4/3} + ...\right]$$

- This is consistent with the time evolution of the entropy density to the first order.

- There is some discrepancy at the second order. However, it does not mean inconsistency immediately.

From Hydro.

$$S = S_\infty\left[1 - \frac{3\eta_0}{2}\tau^{-2/3} + O(\tau^{-4/3})\right] \qquad \eta_0 = \frac{1}{3w}$$

Non-staticity of the loal geometry

Projected Weyl tensor

$$C^{x^1x^2}{}_{x^1x^2} = \frac{w^4}{u^4} - \frac{4w^4}{3u^5}\tau^{-2/3} + ...$$

$$C^{x^1y}{}_{x^1y} = \frac{w^4}{u^4} - \left(\frac{4w^4}{3u^5} + \frac{3\eta_0 w^4}{u^4}\right)\tau^{-2/3} + ...$$

An-isotropy evolves in time.

The dual geometry is not locally static,
if we include dissipation.

What we have done:

- We constructed a consistent gravity dual of the Bjorken flow for the first time.
  (cf. Heller-Loganayagam-Spalinski-Surowka-Vazquez, arXiv:0805.3774)

- Our model is a concrete well-defined example of time-dependent AdS/CFT
  based on a well-controled approximation.

Time evolution of the stress tensor

Hydrodynamics		Our model
• hydrodynamic equation (energy-momentum conservation)	⟺	5d Einstein's eq. at the vicinity of the boundary
• equation of state (conformal invariance)		
• transport coefficients	⟺	Reguarity around the horizon

Related to local thermal equilibrium

Discussion

- The definition of the late-time approximation is a bit artificial.

($\tau^{-2/3}$ expansion with $r\tau^{1/3} = u$ fixed.)

We are trying to "derive" it purely within the gravity theory.

Attractor of the differential equation?

Discussion

- At this stage, the connection among our method and other methods are not clear.

  - Kubo formula:

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int d^4 x \, e^{i\omega t} \left\langle \left[T_{xy}(t, x), T_{xy}(0,0) \right] \right\rangle$$

  - Quasi normal modes

In this case, we impose the “ingoing boundary condition” at the horizon.

⇕

regularity?

My hope

Einstein + Penrose > Kubo + Landau

久保亮五

I am glad if string (gravity) theory can say something nontrivial to

- hydrodynamics
- QGP
- non-equilibrium physics

- plasma instability
- turbulence
- …….

Future directions

- Inclusion of R-charge.
- Can we observe plasma instability?

- What happens if we take τ small enough?
  - Breakdown of the late-time approximation.
  - Breakdown of local thermal equilibrium?
  - Appearance of turbulence?
  - ……..

- A similar analysis based on the Sakai-Sugimoto model.

A message

- We have a very good, challenging problem which is connected the experiments at RHIC and LHC.
- The theoretical framework is deeply overlapped with nuclear science, string theory, general relativity, fluid dynamics, and perhaps with non-equilibrium statistical physics.

Now it is time to collaborate beyond the research fields.

Supplement

Some thought on the regularity

Cosmic censorship hypothesis: (Penrose, 1969)

Naked singularity is not created by any physical pr…

Singularity which is not covered by the event horizon.

The “plasma” w… geometry with a naked singularity is not created by any physical process of the YM theory.

⇕

If you find a naked singularity, such a plasma (with your parameter) cannot be realized by any physical process.

Various quantities from the geometry

Stefan-Boltzmann: $\rho = \frac{3}{8}\pi^2 N_c^2 T_H^4(\tau)$

Entropy creation: Numerical coefficient is given.

$$S = \frac{A}{4G} = \left(\frac{N_c{}^2}{2\pi}\right)^{1/4}\left(\frac{\pi}{3}\right)^{3/4} 2\sqrt{2}\rho_0{}^{3/4}\left(1 - \frac{3}{2}\frac{\eta_0}{\rho_0}\tau^{-2/3} + O(\tau^{-4/3})\right)$$

Integration constant is given.

From hydrodynamics:

$$S = S_\infty - 2\frac{\eta_0}{T_0}\tau^{-2/3} + O(\tau^{-4/3})$$

Not only consistent with hydro but also more information in the holographic dual.

Entropy creation

$$S(\tau) = \int d\tau \frac{4\eta}{3\tau T} = S_\infty - 2\frac{\eta_0}{T_0}\tau^{-2/3} + O(\tau^{-4/3})$$

The dissipation creates the entropy.

The entropy (per unit volume on the LRF) at the infinitely far future is not determined in this framework. (Integration constant)

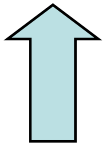
AdS/CFT gives more information.

We need microscopic theory.

Kubo formula:

$$\eta = \lim_{\omega \to 0} \frac{1}{2\omega} \int d^4 x \, e^{i\omega t} \left\langle \left[T_{xy}(t, x), T_{xy}(0,0) \right] \right\rangle$$

We need to compute the two-point function of the current operator.



Obtainable only from the microscopic theory.