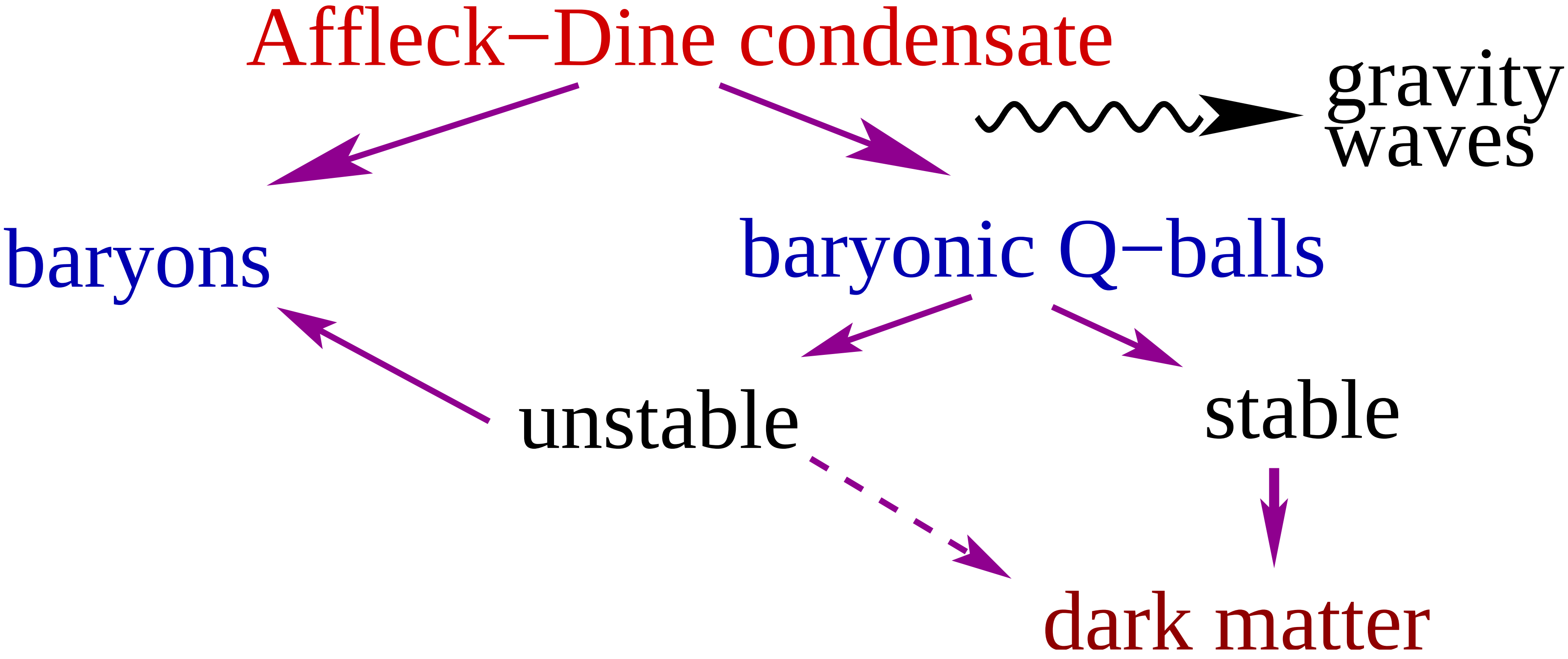
Echoes of supersymmetry: BAU, relic Q-balls, and gravity waves

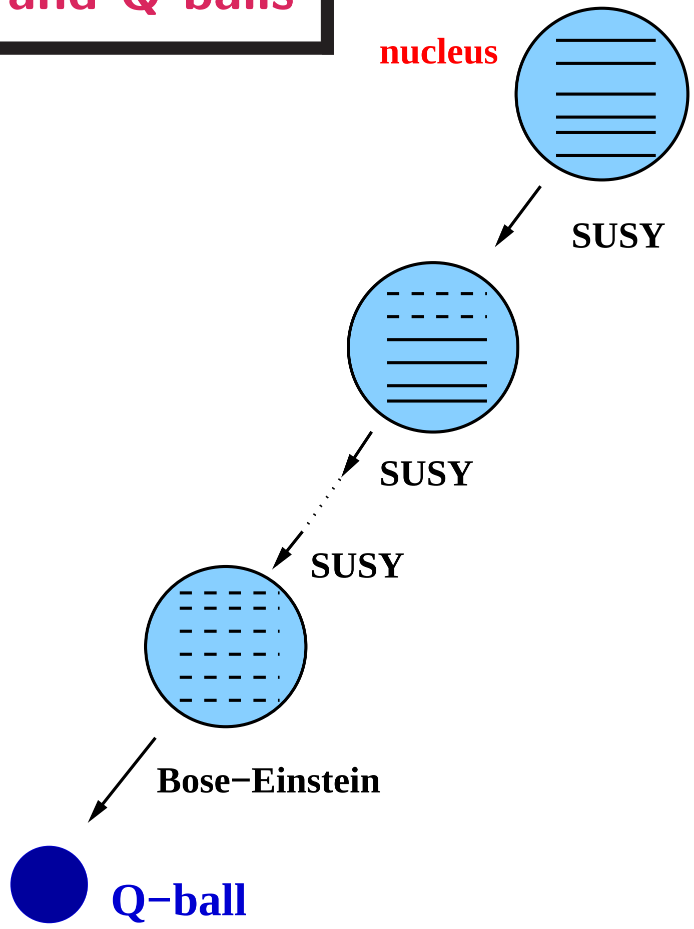
- Introduction: SUSY Q-balls
- Inflation+SUSY⇒ Q-balls
- stable Q-balls as dark matter
- constrains
- gravitational waves

Echoes of supersymmetry: BAU, relic Q-balls, and gravity waves



SUSY and Q-balls

Why would one suspect that SUSY $\Rightarrow$ Q-balls?



Q-balls

Let us consider a complex scalar field $\phi(x, t)$ in a potential that respects a U(1) symmetry: $\phi \longrightarrow e^{i\theta}\phi$.

vacuum: $\phi = 0$

conserved charge: $\boldsymbol{Q = \frac{1}{2i} \int \left(\phi^\dagger \overleftrightarrow{\partial}_0 \phi \right) d^3x}$

$\boldsymbol{Q \neq 0 \Rightarrow \phi \neq 0}$ in some finite domain

$\Rightarrow$ Q-ball [Rosen; Friedberg, Lee, Sirlin; Coleman]

Minimize energy $E = \int d^3x \left[\frac{1}{2}|\dot{\phi}|^2 + \frac{1}{2}|\nabla\phi|^2 + U(\phi)\right]$ under the constraint $Q = \mathrm{const.}$ Introduce Lagrange multiplier:

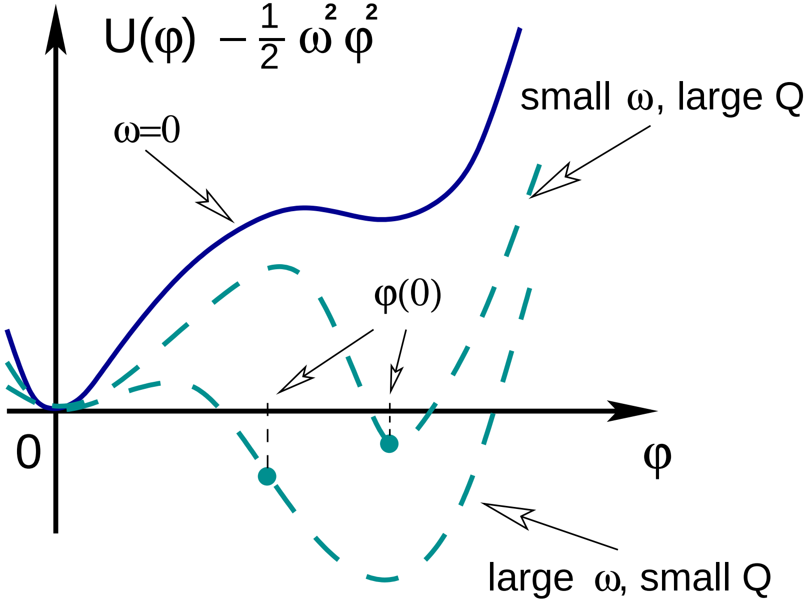
$$\mathcal{E} = E + \omega \left[Q - \frac{1}{2i}\int \phi^* \overleftrightarrow{\partial}_t \, \phi \, d^3x\right]$$

$$= \int d^3x \, \frac{1}{2}\left|\frac{\partial}{\partial}\phi - i\omega\phi\right|^2 + \int d^3x \left[\frac{1}{2}|\nabla\phi|^2 + \hat{U}_\omega(\phi)\right] + \omega Q,$$

where $\hat{U}_\omega(\phi) = U(\phi) - \frac{1}{2}\omega^2\phi^2$.

- Minimize blue by setting $\phi = e^{i\omega t}\bar{\phi}(x)$
- Minimize red by choosing $\bar{\phi}(x)$ to be the **bounce for tunneling in** $\hat{U}_\omega(\phi) = U(\phi) - \frac{1}{2}\omega^2\phi^2$.
- Finally, minimize $\mathcal{E}$ with respect to ω.

Q-balls exist whenever $\hat{U}_\omega(\phi) = U(\phi) - \frac{1}{2}\omega^2\phi^2$ is not positive definite for some value of ω.



Q-balls exist if

$$\boxed{U(\phi)\,/\phi^2 = \min, \quad \text{for } \phi = \phi_0 > 0}$$

[Coleman]

Finite ϕ_0: $M(Q) \propto Q$

Flat potential ($U(\phi) \sim \phi^p, p < 2$); $\phi_0 = \infty$:

$$\boxed{M(Q) \propto Q^\alpha, \alpha < 1}$$

Q-balls exist in (softly broken) SUSY because

- the theory has scalar fields

- the scalar fields carry conserved global charge (baryon and lepton numbers)

- attractive scalar interactions (tri-linear terms, flat directions) force $\left(U(\phi)/\phi^2\right) = \min$ for non-vacuum values.

MSSM, gauge mediated SUSY breaking

Baryonic Q-balls (B-balls) are entirely stable if their mass per unit baryon charge is less than the proton mass.

$$M(Q) = M_S Q^{3/4} \Rightarrow$$

$$\frac{M(Q_B)}{Q_B} \sim M_S Q^{-1/4} \; < 1\mathrm{GeV}$$

$$\text{for } Q_B \gg \left(\frac{M_S}{1\mathrm{GeV}}\right)^4 \gtrsim 10^{12}$$

Such B-balls are entirely stable.

Baryon asymmetry

$$\eta \equiv \frac{n_B}{n_\gamma} = \left(6.1\,^{+0.3}_{-0.2}\right) \times 10^{-10} (\mathrm{WMAP})$$

What happened right after the Big Bang?

- Inflation probably took place
- Baryogenesis – definitely *after* inflation

Standard Model is not consistent
with the observed baryon asymmetry (assuming inflation)

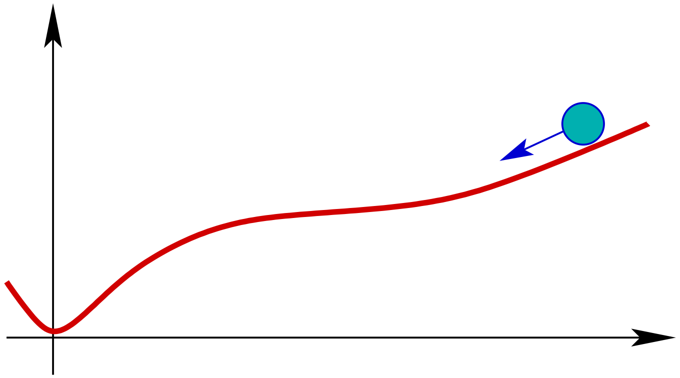
Affleck–Dine baryogenesis

- Natural if SUSY+Inflation
- Can explain matter
- Can explain **dark** matter
- Predictions can be tested soon

Inflation

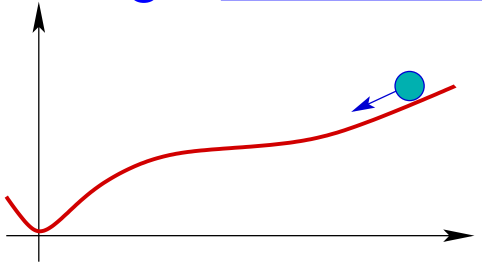
All matter is produced during reheating after inflation.

SUSY ⇒ flat directions.
During inflation, scalar fields
are displaced from their minima.



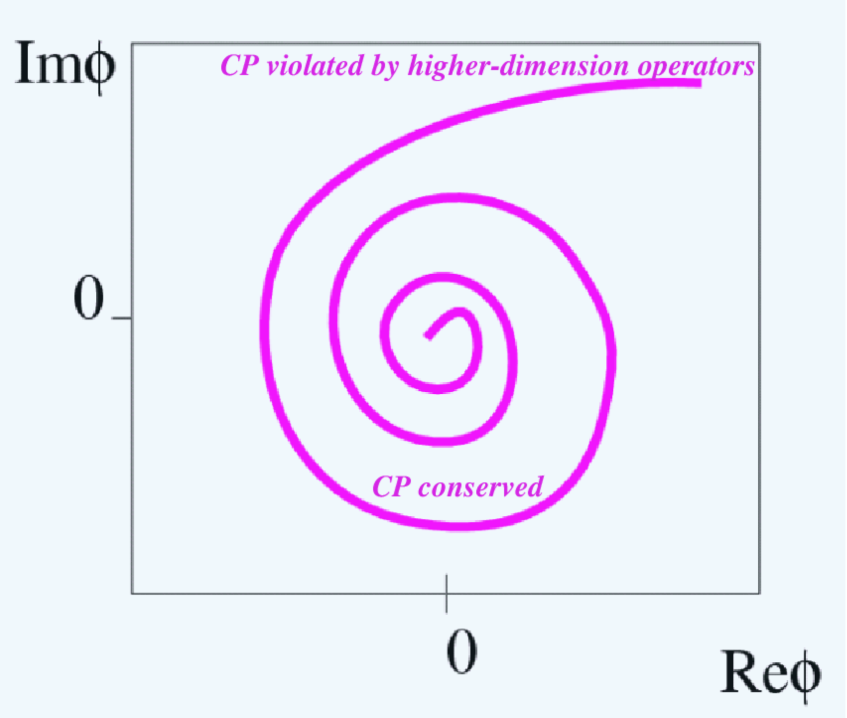
Affleck – Dine baryogenesis

at the end of inflation
a scalar condensate
develops a large VEV
along a **flat direction**



CP violation is due to time-dependent background.

Baryon asymmetry: $\phi = |\phi| e^{i\omega t}$



Affleck – Dine baryogenesis: an example

Suppose the flat direction is lifted by a higher dimension operator $W_n = \frac{1}{M^n}\Phi^{n+3}$. The expansion of the universe breaks SUSY and introduces mass terms $m^2 \sim \pm H^2$.

The scalar potential:

$$V = -H^2|\Phi|^2 + \frac{1}{M^{2n}}|\Phi|^{2n+4}$$

Assume the **inflation scale** $E \sim 10^{15}$ **GeV** The Hubble constant $H_I \approx E^2/M_p \approx 10^{12}$ **GeV**. $T_R \sim 10^9$ **GeV**

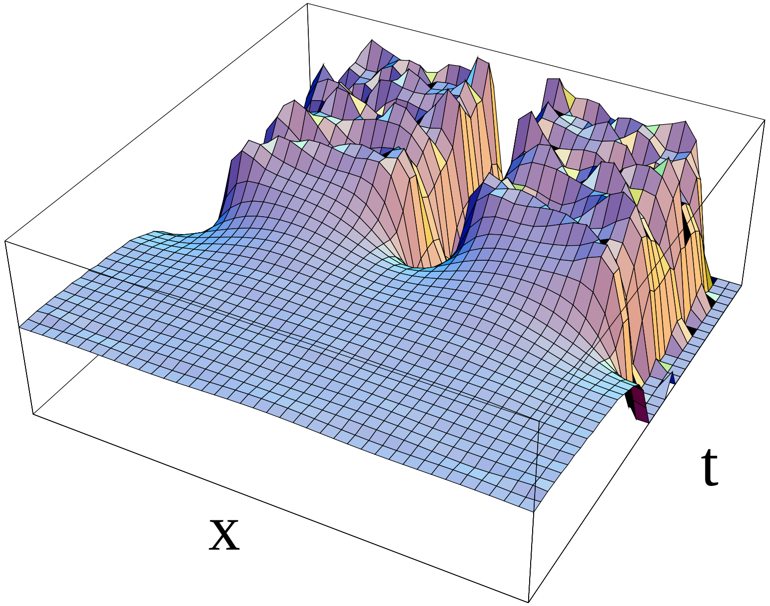
In this example, the final baryon asymmetry is

$$\frac{n_B}{n_\gamma} \sim \frac{n_B}{(\rho_I/T_R)} \sim \frac{n_B}{n_\Phi}\frac{T_R}{m_\Phi}\frac{\rho_\Phi}{\rho_I}$$

$$\sim 10^{-10}\left(\frac{T_R}{10^9\mathrm{GeV}}\right)\left(\frac{M_p}{m_{3/2}}\right)^{\frac{(n-1)}{(n+1)}}$$

Correct baryon asymmetry for $n = 1$. (For $n > 1$, too big.)

Fragmentation of the Affleck-Dine condensate



[AK, Shaposhnikov]

small inhomogeneities can grow

unstable modes:

$$0 < k < k_{\max} = \sqrt{\omega^2 - U''(\phi)}$$

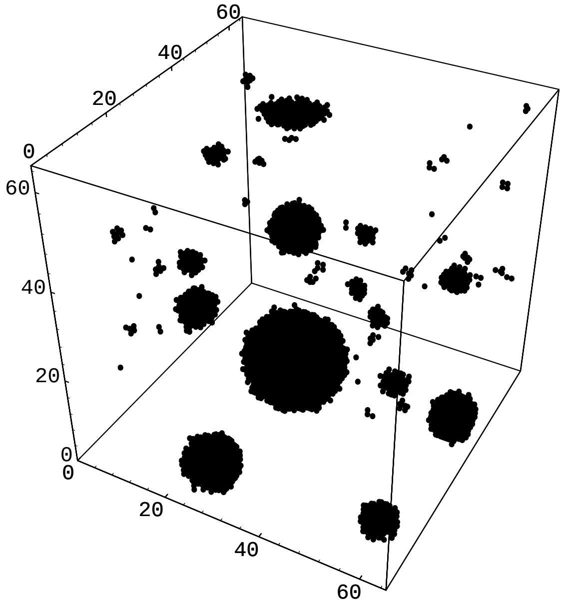
⇒ **Lumps of baryon condensate**

⇒ **Q-balls**

Fragmentation ≈ pattern formation

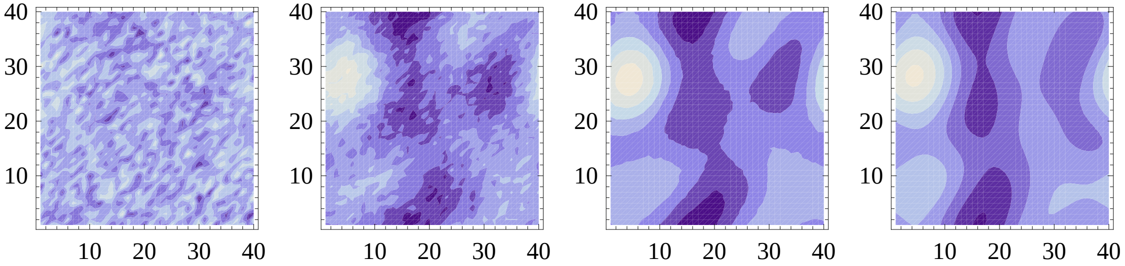
Familiar example:

Numerical simulations of the fragmentation

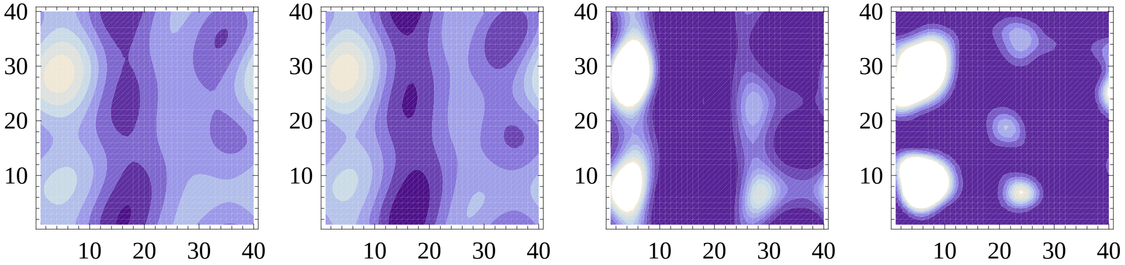


[Kasuya, Kawasaki]

Two-dimensional charge density plots [Multamaki].

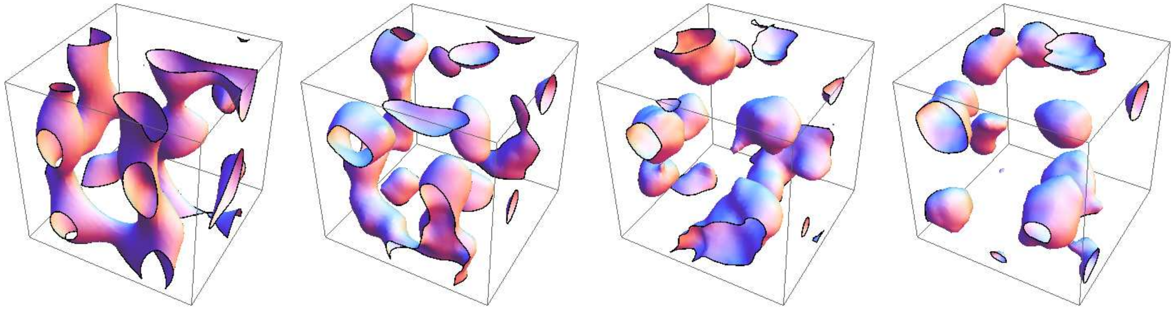


(a) $mt = 0$ (b) $mt = 75$ (c) $mt = 150$ (d) $mt = 375$



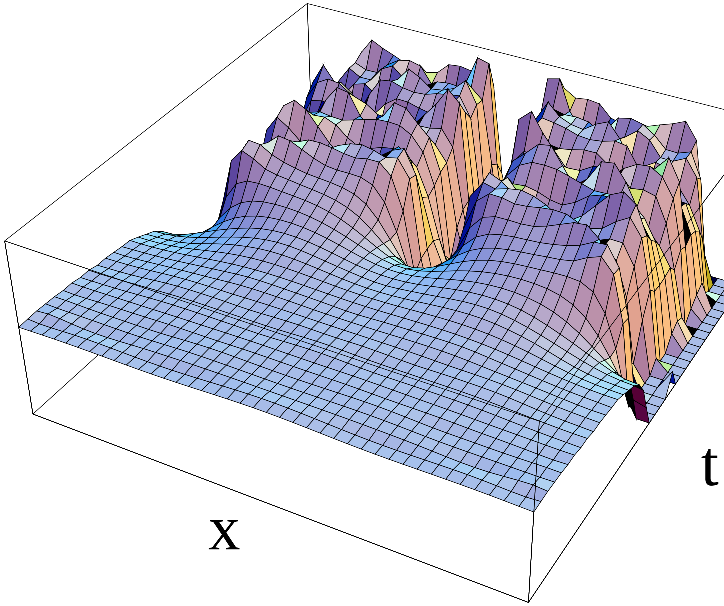
(e) $mt = 525$ (f) $mt = 675$ (g) $mt = 825$ (h) $mt = 900$

Three-dimensional charge density plots [Multamaki].



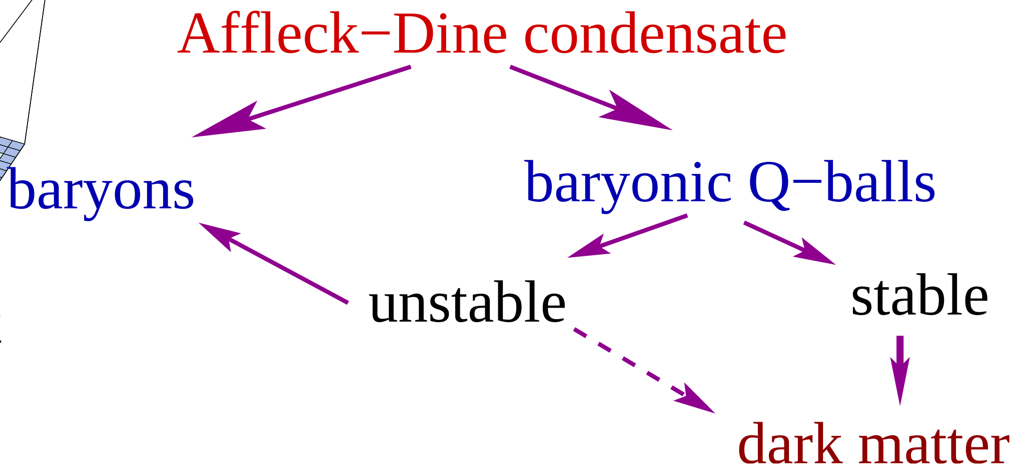
(i) $mt = 900$ (j) $mt = 1050$ (k) $mt = 1200$ (l) $mt = 1350$

Fragmentation of AD condensate can produce Q-balls



SUSY Q-balls may be stable or unstable

if stable ⇒ **dark matter**



Affleck−Dine condensate

baryons

baryonic Q−balls

unstable

stable

dark matter

[AK, Shaposhnikov; Enqvist, McDonald]

Stable Q-balls as dark matter

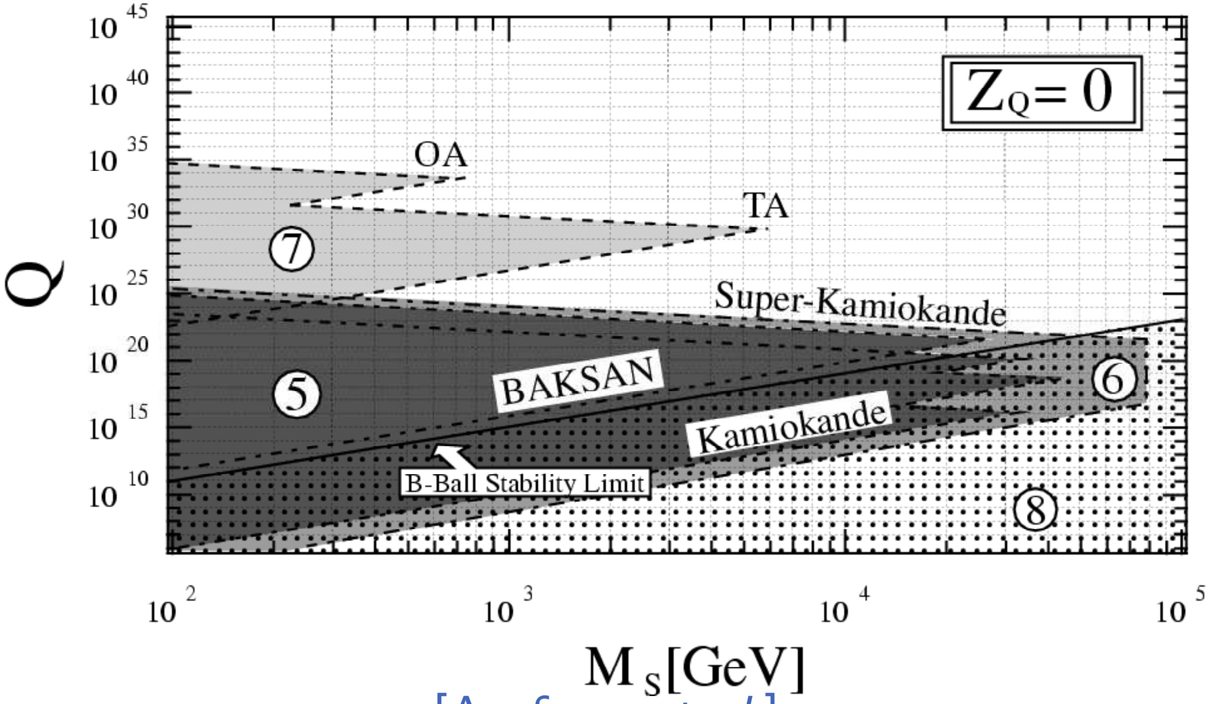
Q-balls can accommodate baryon number at lower energy than a nucleon $\Rightarrow$ **B-Balls catalyze proton decay** Signal:

$$\boxed{\frac{dE}{dl} \sim 100 \left(\frac{\rho}{1\,\mathrm{g/cm^3}}\right) \frac{\mathrm{GeV}}{\mathrm{cm}}}$$

Heavy ⇒ low flux

⇒ **experimental limits from Super-Kamiokande and other *large* detectors**

Present experimental limits



[Arafune *et al.*];

A “candidate event”

C.M.G. Lattes et al., Hadronic interactions of high energy cosmic-ray observed by emulsion chambers

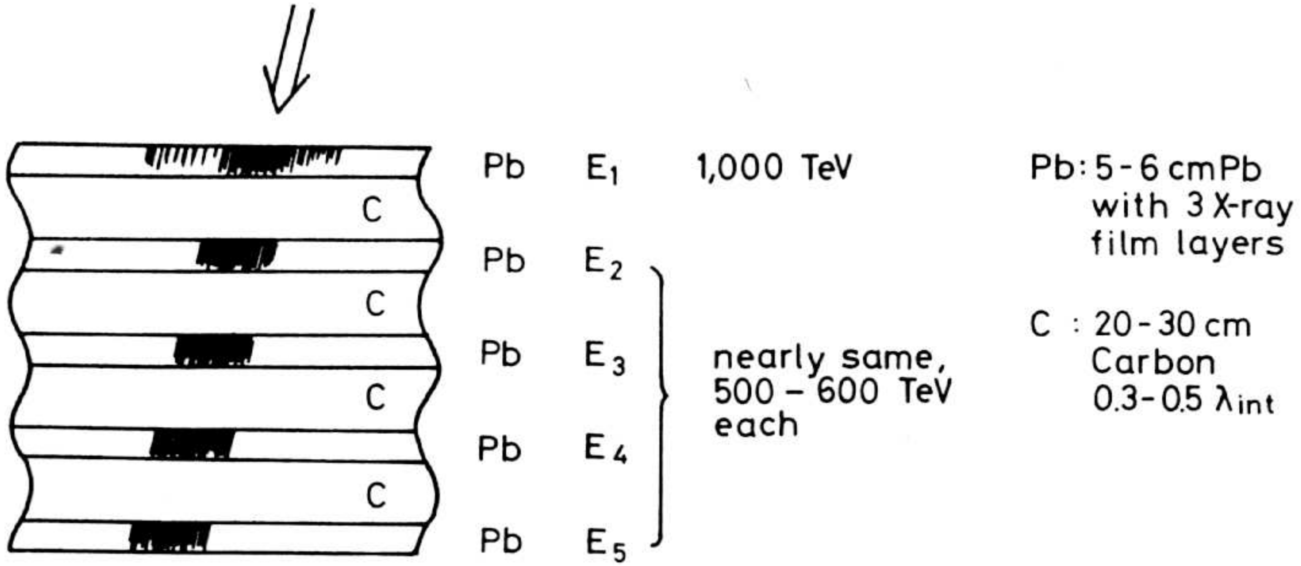


Fig. 47. Illustration of penetrating cores of Pamir experiment.

[Lattes, Fujimoto and Hasegawa, Phys.Rept. **65**, 151 (1980)]

Unstable B-balls

Gravity mediated SUSY breaking typically produces potentials which grow as $\sim \phi^2$ up to the Planck scale.

Hence, *Q-balls are unstable*.

Decay of Q-balls results in *late non-thermal production of LSP.*

Ordinary and dark matter arise from the same process. Hence, one may be able to **explain why** $\Omega_{\rm matter}$ **and** $\Omega_{\rm dark}$ **are not very different**. [Fijii,Yanagida; Enqvist, McDonald; Laine, Shaposhnikov]

$\Omega_{\rm dark}/\ \Omega_{\rm matter} \sim 10$

- Dark matter is **stable Q-balls** [Laine, Shaposhnikov]
- Dark matter is **LSP** produced non-thermally from decay of unstable Q-balls [Enqvist, McDonald; Fujii, Hamaguchi; Fujii, Yanagida]
- Dark matter is **gravitino** produced non-thermally from decay of unstable Q-balls [Fujii, Yanagida]

$\boldsymbol{\Omega}_{\mathbf{B-ball}} / \ \boldsymbol{\Omega}_{\mathbf{matter}} \sim 10$

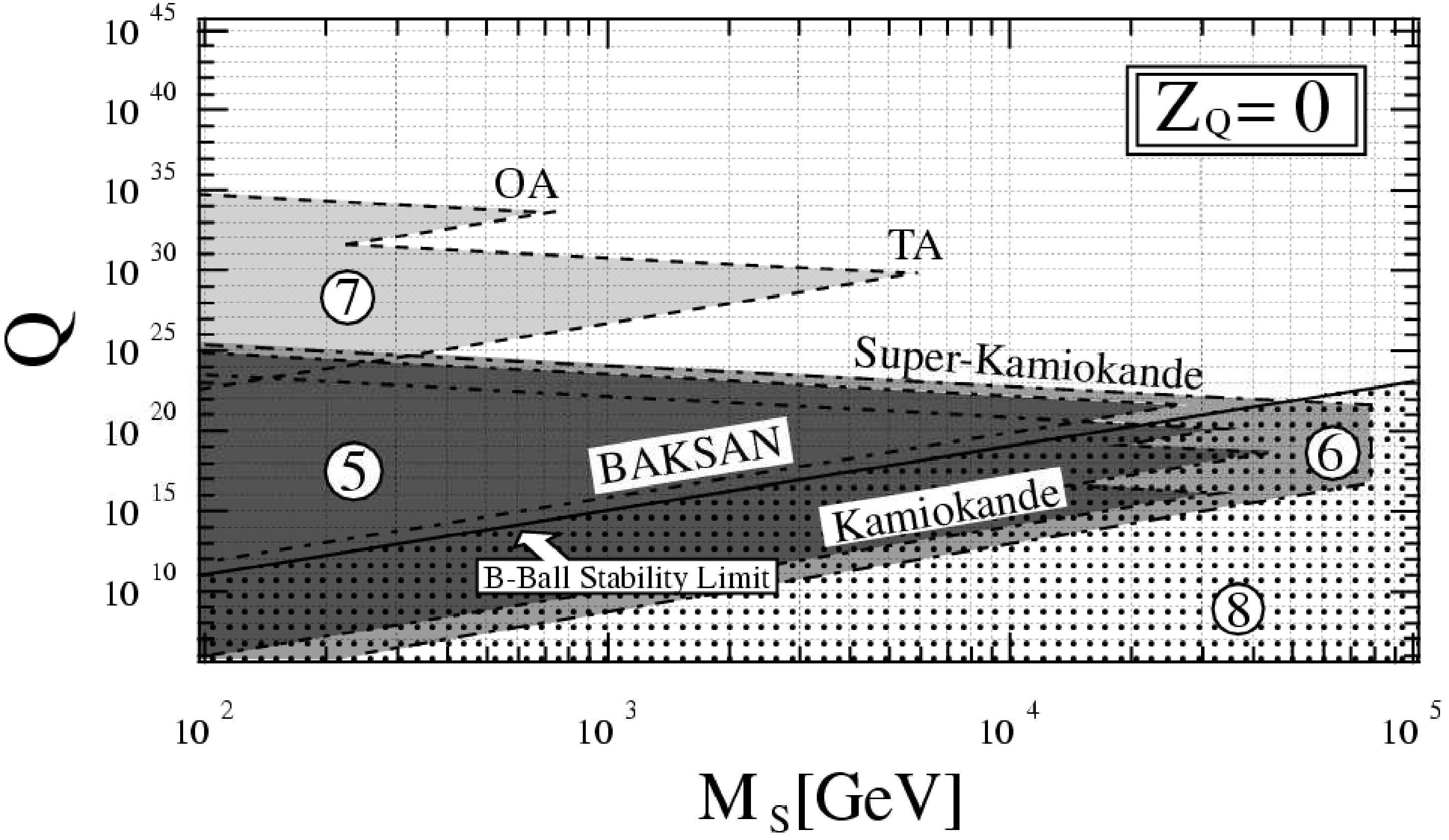
[Laine, Shaposhnikov]

- Gauge-mediated SUSY breaking
- $\boldsymbol{Q}_{\mathrm{B}} \sim \mathbf{10^{26\pm 2}}$ (in agreement with numerical simulations)

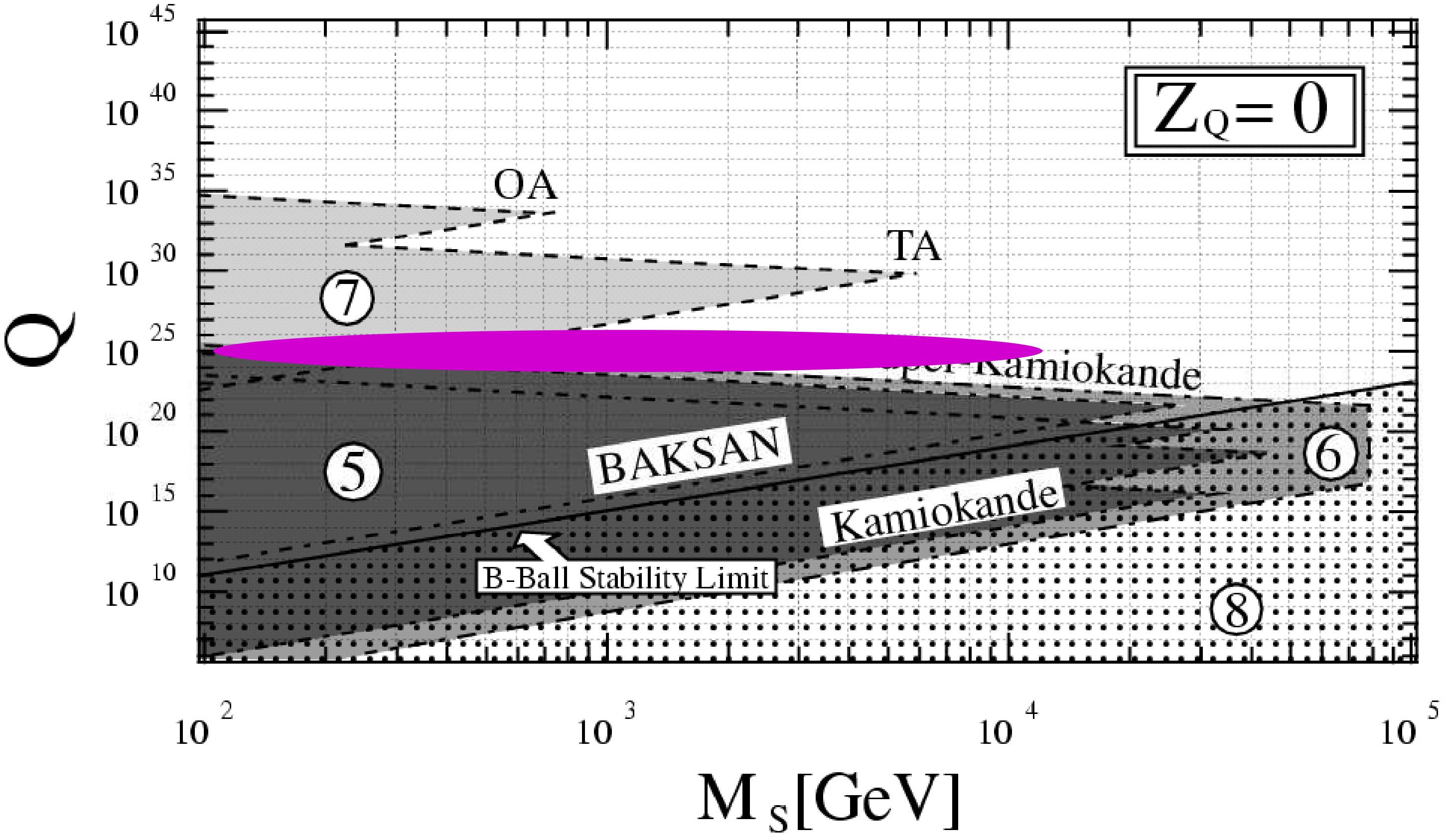
More specifically, $\boldsymbol{\Omega}_{\mathbf{B-ball}}/\Omega_{\mathbf{matter}} \sim 10$ implies

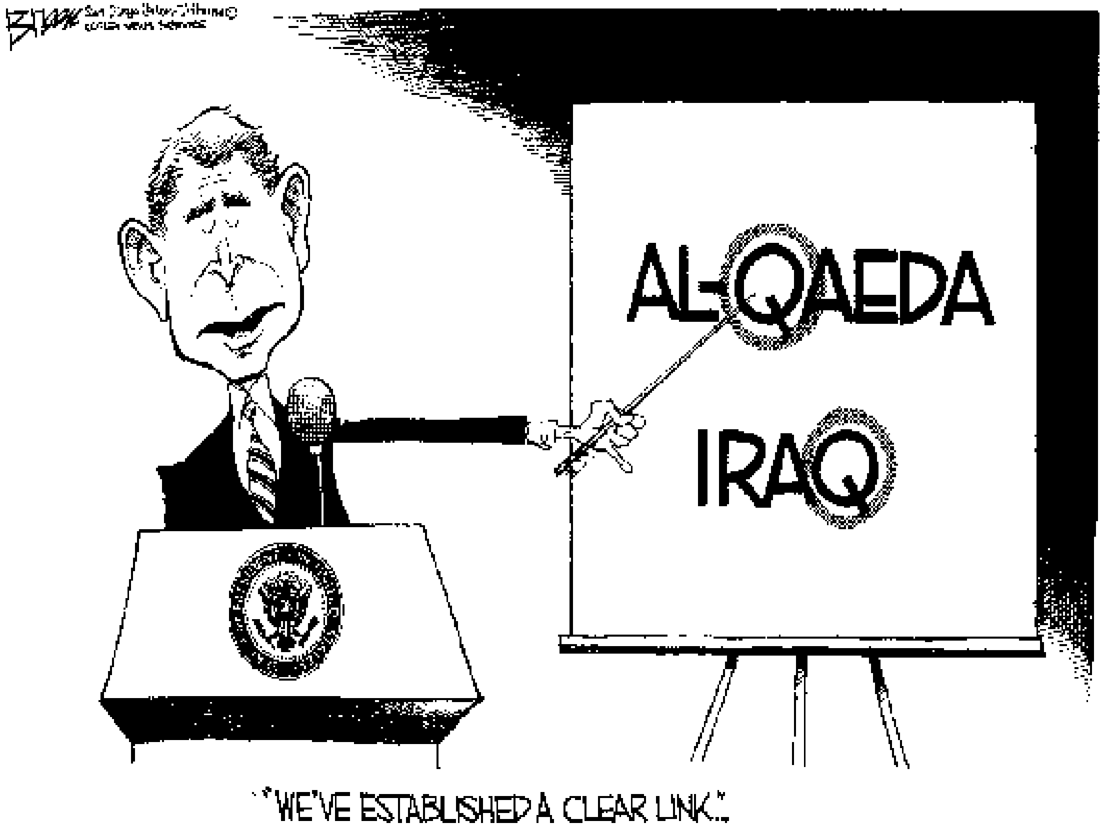
$$\eta_{\mathrm{B}} \sim \mathbf{10^{-10}} \left(\frac{M_{\mathrm{SUSY}}}{\mathrm{TeV}} \right) \left(\frac{Q_{\mathrm{B}}}{\mathbf{10^{26}}} \right)^{\mathbf{-1/2}}$$

$\Omega_{B-ball}/\ \Omega_{matter} \sim 10$

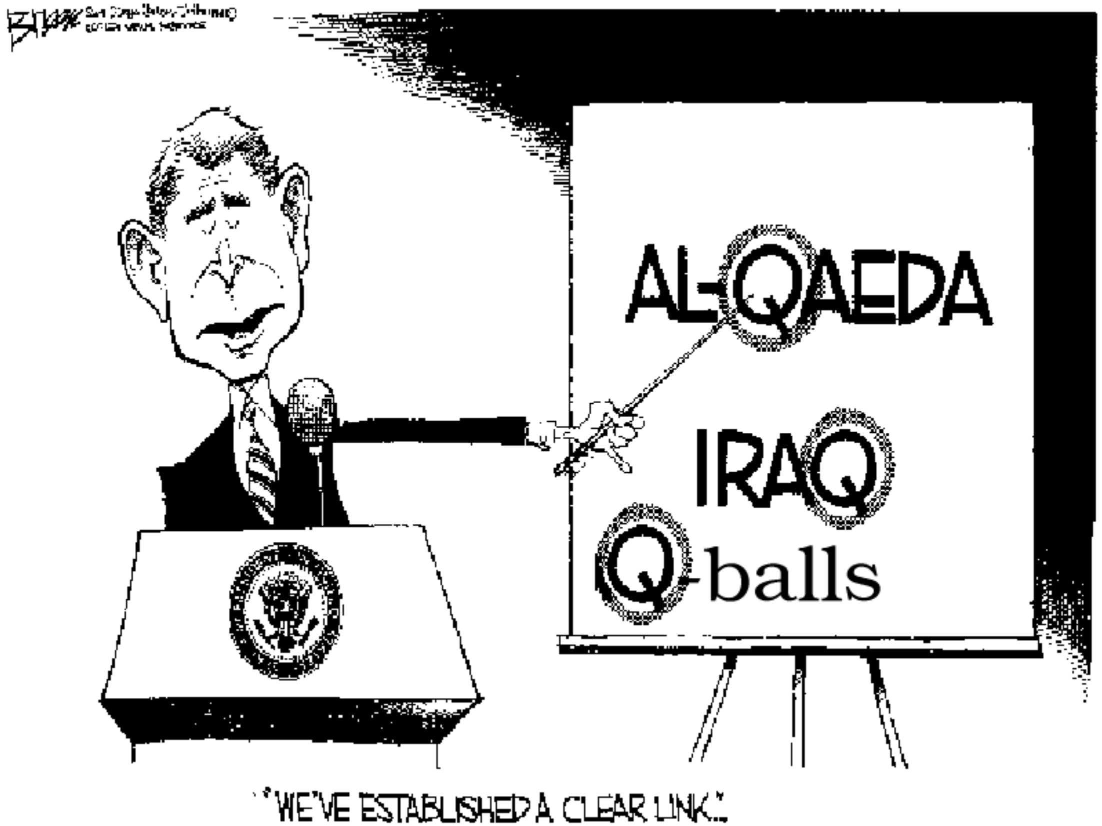


$$\boxed{\Omega_{\rm B-ball}/\ \Omega_{\rm matter} \sim 10}$$





"We've established a clear link"

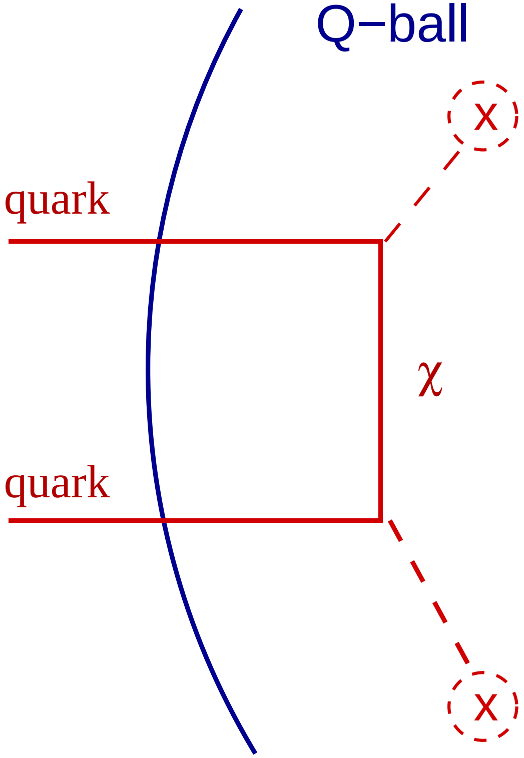


"We've established a clear link"

Astrophysical constraints

- Q-balls pass through ordinary stars and planets
- SUSY Q-balls accumulate inside white dwarfs and neutron stars
- SUSY Q-balls can convert nuclear matter into squark condensate
  - old estimates underestimated the rates
  - new rates too high, unless the **flat direction is lifted by baryon number violating operators**.

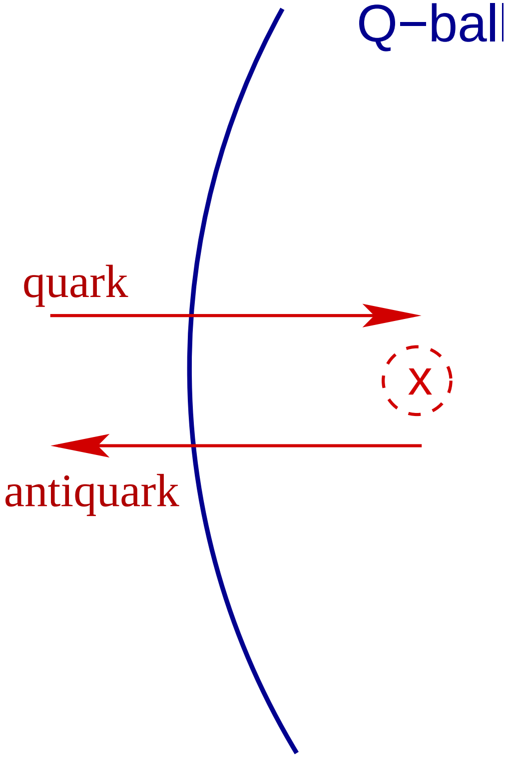
Interactions of SUSY Q-balls with matter (old picture)



$\propto \frac{1}{m_\chi^2}$, slow

This process was thought to limit the rate at which the Q-balls could process baryonic matter. Lifetimes of neutron stars were though to be greater than the age of the universe

Interactions of SUSY Q-balls with matter (correct picture)



There is a Majorana mass term for quarks inside coming from the quark-squark-gluino vertex. **Probability ~ 1 for a quark to reflect as an antiquark.** Very fast!
[AK, Loveridge, Shaposhnikov].

Interactions of SUSY Q-balls with matter

The MSSM Lagrangian contains terms describing interactions of quarks ψ with squarks ϕ and gluinos λ:

$$\mathcal{L} = -g\sqrt{2}T^a_{ij}(\lambda^a \sigma^2 \psi_j \phi^*_i) + C.C. + ...$$

and also the Majorana mass terms for gluinos:

$$\mathcal{L}_{\mathcal{M}} = M\lambda_a\lambda_a.$$

Of course, the quarks also have dirac mass terms.

In the basis $\{\psi_L, \psi_R, \lambda\}$, the mass matrix has a (simplified) form:

$$\left(\begin{array}{ccc} \mathbf{0} & \boldsymbol{m} & \boldsymbol{\varphi_L} \\ \boldsymbol{m} & \mathbf{0} & \boldsymbol{\varphi_R} \\ \boldsymbol{\varphi_L} & \boldsymbol{\varphi_R} & \boldsymbol{M} \end{array} \right)$$

The squark fields ϕ grow large inside the Q-ball. This mass term causes a quark to scatter off a Q-ball as an antiquark, with probability of order 1.

[AK, Loveridge, Shaposhnikov]

Interaction rates are not limited by weak-scale cross section.

Signatures in detectors do not change significantly

Neutron stars: can they survive long enough?

Pulsars ages: oldest pulsars have $(\dot{P}/P) \sim (0.3 - 3) \times 10^{-10} \mathrm{yr}^{-1}$

Some pulsars are also known to be (at least) as old as **10 Gyr** based on the cooling ages of their white dwarf companions

Inside a neutron star Q-ball VEV grows fast and reaches vealues at which the flat direction is lifted by higher-dimension operators

Generally, the lifting terms can be written in the form

$$V^n(\phi)_{\text{lifting}} \approx \lambda_n M^4 \left(\frac{\phi}{M}\right)^{n-1+m} \left(\frac{\phi^*}{M}\right)^{n-1-m}$$

- If $m \neq 0$, the baryon number is broken. Q-balls inside a neutron star reach some maximal size and stop growing in size. The rate of conversion of matter into condensate stabilizes at a small value. This is allowed.

- If $m = 0$, Q-balls change the way they grow after reaching a certain size Q_c.

Q-balls along "Flat" and "Curved" directions

	FD	CD
φ	$\frac{1}{\sqrt{2}}\Lambda Q^{1/4}$	$\varphi_{\max}$
ω	$\pi\sqrt{2}\Lambda Q^{-1/4}$	$\Lambda^2\varphi_{\max}^{-1} = \pi\sqrt{2}\Lambda {Q_c}^{-1/4} = \omega_c$
M	$4\pi\frac{\sqrt{2}}{3}\Lambda Q^{3/4}$	ωQ
R	$\frac{1}{\sqrt{2}\Lambda}Q^{1/4}$	$\left(\frac{3}{8\pi}\frac{1}{\Lambda^2\varphi_{\max}}Q\right)^{1/3} = (\frac{3}{2})^{1/3}(Q/Q_c)^{1/12}R_{FD}$

The change from FD to CD makes the Q-ball grow faster.

As soon as this happens, the neutron star is consumed very quickly:

	FD Q-balls	CD Q-balls
t	$\mathbf{10^{10}}$ **years**	$\mathbf{1500}$ **years**

Q-balls that go from FD to CD for $\boldsymbol{Q < 10^{57}}$ are ruled out, unless the lifting terms can break the baryon number.

White Dwarfs

White dwarfs can also accumulate SUSY Q-balls. The rate of consumption is lower because of the lower density. Nevertheless, one should consider a possible limit coming from the fact that some very old (10 Gyr) white dwarfs are known to have cooled down to very low temperatures; they emit

$$\boldsymbol{L_{\rm wd} = 3 \times 10^{-5} L_\odot = 7 \times 10^{28}\,{\rm erg/s}.}$$

Q-balls must not produce more heat than this.

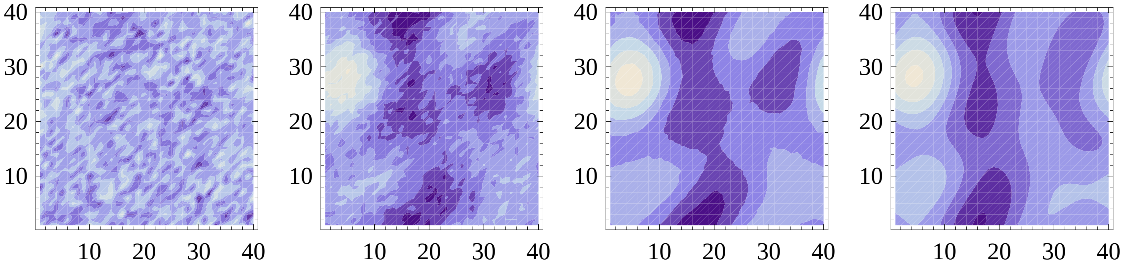
No new limits arise. For $m = 0$, Q-balls are ruled out by stability of neutron stars. For $m \neq 0$, the rate of heat release is much than $L_{\rm wd}$.

Gravitational radiation from the fragmentation process

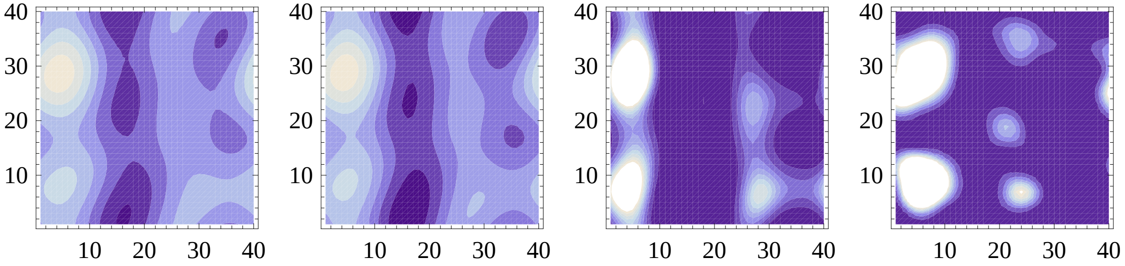
One can expect gravitational waves if

- large masses move around
- relativistic velocities
- no spherical symmetry

All of these conditions can be satisfied for *some* flat directions.

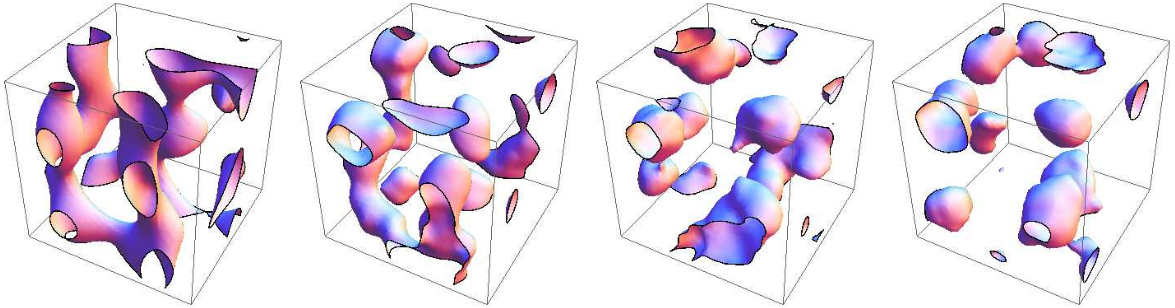


(m) $mt = 0$ (n) $mt = 75$ (o) $mt = 150$ (p) $mt = 375$



(q) $mt = 525$ (r) $mt = 675$ (s) $mt = 825$ (t) $mt = 900$

Two-dimensional charge density plots.



(u) $mt = 900$ (v) $mt = 1050$ (w) $mt = 1200$ (x) $mt = 1350$

Three-dimensional charge density plots.

The lack of spherical symmetry in the early steps of fragmentation means gravity waves can be produced.

Analytical estimates [AK, Mazumdar]

The mass density of the condensate undergoing fragmentation can be written as $\rho(x,t) = \rho_0 + \rho_1(x,t)$, where

$$\rho_1(x,t) = \epsilon\rho_0 \int d^3k \, e^{\alpha_k t} \cos(\omega t - \vec{k} \cdot \vec{x}) \, .$$

The quadrupole moment that generates gravity waves:

$$D_{ij} = \int d^3x \; x_i x_j \, T^{00}(x,t) \, ,$$

where the energy-momentum tensor $T^{00}(x,t) \approx \rho(x,t)$.

Based on the analytical and numerical calculations of the condensate fragmentation [Kawasaki et al.],

$$\boldsymbol{k \sim \xi_k \times 10^2 H_*, \ \omega_k \sim vk \sim \xi_k \times 10^2 \, vH_*},$$

where H_* is the Hubble constant at the time of the condensate.

For $\omega \sim 10^2 vH_*$, the power in gravitational waves in a Hubble volume:

$$\boldsymbol{P \sim 10^4 \xi_k^{-2} \, G \frac{\rho_0^2 v^6}{H_*^4}} \, .$$

For mode $\phi(x,t) \approx R(t) \exp\{\alpha_k t\} \cos(\omega_k t - kx)$, where $R(t)$ is a slowly changing function of time, At the time of production [AK, Mazumdar],

$$\Omega_{GW*} \sim 10^{-3}\xi_k^{-3}\xi_v^{6}\,\frac{\rho_0^2}{(H_* M_{\rm Pl})^4}$$

The energy density depends on the type of SUSY breaking and the type of flat direction.

Strong gravitational waves:

- gravity mediated SUSY breaking (more mass per scalar)
- not the flat direction of AD baryogenesis: $\eta_B = n_B/n_\gamma \sim 10^{-10}$ too small
- $(B+L)$ flat directions OK: sphalerons destroy $(B+L)$, so there is no constraint on the initial density carried by the $(B+L)$ flat directions.

Predictions:

Peak frequency of the gravitational radiation observed today, $f_* = \omega_k/2\pi$:

$$\boldsymbol{f = f_* \frac{a_*}{a_0} = f_* \left(\frac{a_*}{a_{\rm rh}}\right) \left(\frac{g_{s,0}}{g_{s,\rm rh}}\right)^{1/3} \left(\frac{T_0}{T_{\rm rh}}\right)}$$

$$\boldsymbol{\approx 0.6\,\mathrm{mHz}\ \xi_k \xi_v \left(\frac{g_{s,\rm rh}}{100}\right)^{1/6} \left(\frac{T_{\rm rh}}{1\ \mathrm{TeV}}\right) \left(\frac{f_*}{10 H_*}\right)},$$

$T_{\rm rh} \sim 1$ TeV $\Rightarrow$ mHz frequency, accessible to LISA $T_{\rm rh} \sim 100$ TeV $\Rightarrow$ 10–100 Hz frequency, accessible to LIGOIII and BBO

Spectral signature: signal is peaked at the longest wavelength, determined by the size of the Q-balls, and it falls off as $\boldsymbol{1/f^3}$ for larger frequencies.

The fraction of the critical energy density ρ_c stored in the gravity waves today is

$$\begin{aligned}\boldsymbol{\Omega_{\mathrm{GW}}} &= \Omega_{\mathrm{GW}*}\left(\frac{a_*}{a_0}\right)^4\left(\frac{H_*}{H_0}\right)^2 \\ &\approx \frac{1.67\times 10^{-5}}{h^2}\left(\frac{100}{g_{s,*}}\right)^{1/3}\Omega_{\mathrm{GW}*} \approx \boldsymbol{10^{-8}\,\xi_k^{-3}\xi_v^6\,h^{-2}}\end{aligned}$$

LISA sensitivity: $\Omega_{\mathrm{GW}}h^2 \sim 10^{-11}$ at mHz frequencies

LIGO III sensitivity: $\Omega_{\mathrm{GW}}h^2 \sim (10^{-5} - 10^{-11})$ in the $(5 - 10^3)$ Hz frequency band.

Numerical simulations under way [AK, Mazumdar, Multamäki]

Conclusion

- SUSY + Inflation ⇒ Q-balls, some may be stable, may be dark matter
- Typical size large ⇒ typical density small ⇒ need large detectors to search for relic Q-balls
- Gravitational waves from the fragmentation of $(B + L)$ flat directions may be observed by LIGO III and LISA, as well as BBO.