Cosmological Unification of String Theories

Simeon Hellerman

based on :

hep-th/0405041, S.H.
hep-th/0409071, S.H. and Xiao Liu
hep-th/0611317, S.H. and Ian Swanson
hep-th/0612051, S.H. and Ian Swanson
hep-th/0612116, S.H. and Ian Swanson
arXiv:0705.0980, S.H. and Ian Swanson
arXiv:0709.2166, S.H. and Ian Swanson
arXiv:0710.1628, S.H. and Ian Swanson

LPTHE, Paris, 5 Feb 2008

Outline

- Overview of quintessent cosmology and linear dilaton backgrounds
- Supercritical string theory: spacetime effective action
- Stability in time-dependent backgrounds
- Bosonic string solutions with nonzero tachyon
- Dimension-changing solutions in the type 0 string
- Transitions from type 0 to bosonic string theory
- Conclusions

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These cosmologies connect supercritical string theories to the more familiar string duality web in ten dimensions.

They also provide a precise link between supersymmetric and purely bosonic string theory.

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The equation of motion for the scale factor $a(t)$:

$$\frac{\ddot{a}}{a} = -\frac{D-3+w(D-1)}{(D-1)(D-2)}\rho$$

where $w \equiv p/\rho$ is the state equation.

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Consider a theory of a real scalar field ϕ with Lagrangian

$$\mathcal{L}_\phi = \frac{1}{\kappa^2}\sqrt{-\det G}\left[\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - \mathcal{V}(\phi)\right]$$

where

$$\mathcal{V}(\phi) \equiv c\exp(\gamma\phi)\,, \qquad c, \gamma > 0$$

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It follows that $\dot{\phi}^2$, H^2 and $\mathcal{V}$ all scale as t^{-2}, so we find the general expressions

$$\begin{aligned} \phi(t) &= \lambda \log(t/t_1) \\ a(t) &= a_0 \left(\frac{t}{t_0} \right)^{\alpha} \end{aligned}$$

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for some α, λ.

This amounts to a direct relation between $\dot{a}$ and a, which can be integrated to yield the following:

$$\begin{aligned} \alpha &= \frac{2}{(1+w)(D-1)} \\ \gamma^2 &= \frac{2(D-1)(w+1)}{D-2} \end{aligned}$$

Linear dilaton background

Because $c > 0$, the energy density ρ is positive, and the cosmological scale accelerates as a function of FRW time if $-1 \leq w < w_{\rm crit}$, where

$$w_{\rm crit} = -\frac{D-3}{D-1}$$

in D spacetime dimensions.

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The precise nature of this singularity and the nature of the asymptotic future $t \to +\infty$, however, depend on the state equation of the cosmology.

Linear dilaton background

We can rewrite the metric in a canonical form for a conformally flat spacetime. We define a new time coordinate $\bar{t}$ via the equation

$$\bar{t} \equiv \left(\frac{(D-1)(1+w)}{(D-1)w+(D-3)} t_0^{\frac{2}{(D-1)(1+w)}} a_0^{-1} \right) t^{\frac{(D-1)w+(D-3)}{(D-1)(1+w)}}$$

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In these coordinates, the metric takes the form

$$ds^2 = \omega(\bar{t})^2 \left[-d\bar{t}^2 + dx^i dx^i \right] = \omega(\bar{t})^2 \left[-d\bar{t}^2 + dr^2 + r^{D-2} d\Omega_{D-2}^2 \right]$$

where we have defined

$$\omega(\bar{t}) \equiv l \left(\frac{(D-1)w+(D-3)}{(D-1)(1+w)} \bar{t} \right)^{\frac{2}{(D-1)w+(D-3)}}$$

$$l \equiv a_0 \left(\frac{a_0}{t_0} \right)^{\frac{2}{(D-1)w+(D-3)}}$$

Linear dilaton background

To construct Penrose diagrams, one ignores the $(D-2)$-sphere fibered over each diagram and conformally compactifies the (r, t) plane using the transformation

$$r \equiv \frac{\sin\chi}{\cos\chi + \cos\tau} \qquad \bar{t} \equiv \frac{\sin\tau}{\cos\chi + \cos\tau}$$

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In these coordinates, the metric on the (r, t) plane becomes

$$\begin{aligned} ds^2 \quad = \quad & \frac{l^2}{4} \frac{(\sin|\tau|)^{2\Delta}}{\left[\cos\left(\frac{\chi+\tau}{2}\right)\cos\left(\frac{\chi-\tau}{2}\right)\right]^{2+2\Delta}} \left| \frac{(D-1)w + (D-3)}{2(D-1)(1+w)} \right|^{2\Delta} \\ & \times(-d\tau^2 + d\chi^2) \end{aligned}$$

with

$$\Delta \equiv \frac{2}{(D-1)w + (D-3)}$$

Linear dilaton background

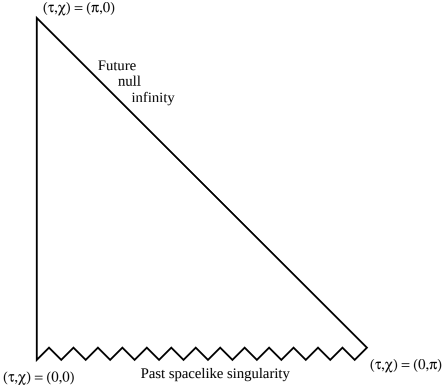
For $-1 < w < w_{\rm crit}$, the constant Δ is negative and the range of the τ and χ coordinates is

$$\tau \in [-\pi, 0] \qquad \chi \in [0, \tau + \pi] \qquad \text{(accelerating universe)}.$$

For $w > w_{\rm crit}$, the quantity Δ becomes positive, and we have

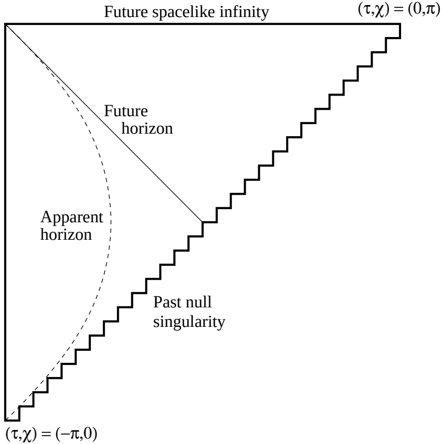
$$\tau \in [0, \pi] \qquad \chi \in [0, \pi - \tau] \qquad \text{(decelerating universe)}.$$

Linear dilaton background



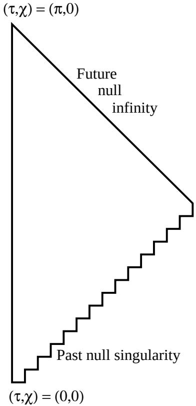
Penrose diagram of the decelerating universe ($w > w_{\rm crit}$). The initial singularity is spacelike, and the future boundary is null.

Linear dilaton background



Penrose diagram of the accelerating $(-1 < w < w_{\rm crit})$ universe. The initial singularity is null, and the future spacelike boundary is obscured from observers by a horizon.

Linear dilaton background



Penrose diagram of the universe with critical equation of state ($w = w_{\rm crit}$).

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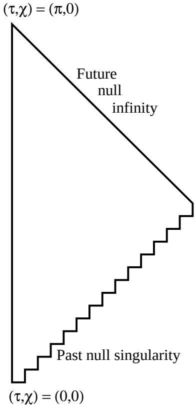

Penrose diagram of the universe with critical equation of state ($w = w_{\mathrm{crit}}$). The initial singularity is null, as is the future boundary. It is conformally equivalent to Minkowski space (conventional big bang, asymptotic infinity, and ordinary final states).

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Supercritical string theory

For the bosonic string in $D > 26$, the effective action for the metric and dilaton appears as

$$S_{\text{eff}} = \frac{1}{2\kappa^2} \int d^D x \sqrt{-\det G^{(S)}} \exp(-2\Phi) \left[-\frac{D-26}{3\alpha'} + R^{(S)} + 4(\partial\Phi)^2 \right]$$

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Higher dimension terms are dropped: such terms in the tree-level action are suppressed by powers of $\alpha' = 1/(2\pi T_{\rm string})$, where $T_{\rm string}$ is the fundamental string tension.

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We may rewrite the action in terms of the Einstein metric using the field redefinition

$$G^{(S)}_{\mu\nu} = \exp\left(\frac{4\Phi}{D-2}\right) G^{(E)}_{\mu\nu}$$

We may also rescale $\Phi \to \frac{1}{2}\sqrt{D-2}\,\phi \ldots$

Supercritical string theory

We obtain:

$$S_{\text{eff}} = \frac{1}{2\kappa^2} \int d^D X \sqrt{-\det G^{(E)}} \left[-\frac{2(D-26)}{3\alpha'} \exp\left(\frac{2\phi}{\sqrt{D-2}} \right) + R^{(E)} - (\partial\phi)^2 \right]$$

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$$\gamma = \frac{2}{\sqrt{D-2}} \qquad c = \frac{D-26}{3\alpha'}$$

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$$S_{\rm eff} = \frac{1}{2\kappa^2}\int d^DX\sqrt{-\det G^{(E)}}\left[-\frac{2(D-26)}{3\alpha'}\exp\left(\frac{2\phi}{\sqrt{D-2}}\right) + R^{(E)} - (\partial\phi)^2\right]$$

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We therefore recover a quintessent solution with equation of state

$$w = -\frac{D-3}{D-1}$$

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Maximal SUSY breaking at weak coupling?

Supercritical string theory

The general quintessent solution is of the form

$$\begin{aligned}\Phi &= \Phi_0 - \frac{D-2}{2}\ln\left(\frac{t}{t_0}\right) \\ a &= \frac{a_0}{t_0}\,t\end{aligned}$$

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$$t_{\rm FRW} = t_0 \exp\left(+\frac{2q\ t_{\rm conf}}{D-2}\right)$$

So when we move to coordinates in which the metric is manifestly conformally flat and we decanonicalize the scalar field, we find that the dilaton is logarithmic as a function of FRW time, and linear as a function of conformal time:

$$ds^2 = \frac{a_0^2}{t_0^2}\ t^2 \left(-dt_{\rm conf}^2 + dx^i dx^i\right) = a^2 \eta_{\mu\nu} dx^\mu dx^\nu \qquad (1)$$

$$\Phi = \Phi_0 - q\ t_{\rm conf}$$

Outline

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Exponentially growing modes will have decreasing effect on the remaining degrees of freedom in the theory if they grow more slowly than g_s^{-1}.

A useful definition, therefore, is that unstable modes must grow faster than g_s^{-1} at late times.

Stability

Consider a massless scalar coupled to the string:

$$\mathcal{L}_\sigma = -\frac{1}{2\kappa^2}\sqrt{-\det G^{(S)}}e^{-2\Phi}(\partial\sigma)^2 = -\frac{1}{2\kappa^2}\sqrt{-\det G^{(E)}}(\partial\sigma)^2$$

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$$\sigma = \sigma_\infty - \xi\, t^{-(D-2)}$$

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where σ_∞ and ξ are constants of motion that can take arbitrary real values.

The modes of the field σ asymptote to the constant value σ_∞ as $t \to \infty$.

From the point of view of the Einstein frame, this effect is due to Hubble friction; in the string frame this behavior is understood to be caused by the drag force arising from the interaction between σ and the linear dilaton.

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This induces a mass term for the rescaled field that represents a proper quantum of string:

$$e^{-2\Phi}(\partial\sigma)^2 = (\partial\tilde{\sigma})^2 + \tilde{\sigma}^2(\partial\Phi)^2 + 2\tilde{\sigma}(\partial\tilde{\sigma})\left[(\partial\Phi)_{\text{background}} + (\partial\Phi)_{\text{fluctuation}}\right]$$

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- The fluctuation term represents a trilinear vertex that we discard.
- The background term is constant, so its product with $\tilde{\sigma}\,\partial\tilde{\sigma}$ amounts to a total derivative.

Stability

The mass term for the rescaled field is tachyonic, and equal to $-q^2$:

$$\mathcal{L}_{\tilde{\sigma}} \sim -\frac{1}{2\kappa^2}\left[(\partial\tilde{\sigma})^2 - q^2\tilde{\sigma}^2\right]$$

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Since $g_s\tilde{\sigma}$ is just σ (the original field appearing in the spacetime action in front of $\exp(-2\Phi)$), the requirement for physical stability is that modes normalized to have the factor $\exp(-2\Phi)$ in their kinetic term should shrink exponentially in the future (or at least remain constant).

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So, canonical modes may grow exponentially in time, but they do not necessarily represent a physical instability: the exponential growth is countered by the shrinking string coupling.

[Hellerman, IS: hep-th/0611317; Aharony, Silverstein: hep-th/0612031]

Stability

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We will now discuss a large class of solvable and exactly marginal perturbations of this form.

Outline

Bubble of nothing

Consider a theory with stress tensor

$$T_{++} = -\frac{1}{\alpha'} : \partial_{\sigma^+} X^\mu \partial_{\sigma^+} X_\mu : + \partial^2_{\sigma^+}(V_\mu X^\mu)$$

$$T_{--} = -\frac{1}{\alpha'} : \partial_{\sigma^-} X^\mu \partial_{\sigma^-} X_\mu : + \partial^2_{\sigma^-}(V_\mu X^\mu)$$

where colons represent normal ordering of the $2D$ theory. Here, $\sigma^\pm$ are particular light-cone combinations of the worldsheet coordinates $\sigma^{0,1}$:

$$\sigma^\pm = -\sigma^0 \pm \sigma^1$$

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$$\sigma^\pm = -\sigma^0 \pm \sigma^1$$

Physical states of the string correspond to local operators $\mathcal{U}$ that are Virasoro primaries of weight one. That is, their operator product expansion (OPE) with the stress tensor satisfies:

$$T_{++}(\sigma)\mathcal{U}(\tau) \simeq \frac{\mathcal{U}(\tau)}{(\sigma^+ - \tau^+)^2} + \frac{\partial_+ \mathcal{U}(\tau)}{\sigma^+ - \tau^+}$$

and similarly for T_{--},

Bubble of nothing

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$$\mathcal{U}_M \equiv : \mathcal{T}(X) :$$

and admits the following on-shell condition:

$$\partial_\mu \partial^\mu \mathcal{T}(X) - 2V^\mu \partial_\mu \mathcal{T}(X) + \frac{4}{\alpha'} \mathcal{T}(X) = 0$$

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For tachyon profiles of the form

$$\mathcal{T}(X) = \mu^2 \exp\left(B_\mu X^\mu\right)$$

this condition is

$$B^2 - 2V \cdot B = -4/\alpha'$$

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and admits the following on-shell condition:

$$\partial_\mu \partial^\mu \mathcal{T}(X) - 2V^\mu \partial_\mu \mathcal{T}(X) + \frac{4}{\alpha'} \mathcal{T}(X) = 0$$

For tachyon profiles of the form

$$\mathcal{T}(X) = \mu^2 \exp\left(B_\mu X^\mu\right)$$

this condition is

$$B^2 - 2V \cdot B = -4/\alpha'$$

A general value of B_μ will lead to a nontrivial interacting theory when the strength μ^2 of the perturbation is treated as non-infinitesimal.

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We therefore put B_μ in the form

$$\begin{aligned} B_0 &= B_1 \equiv \beta/\sqrt{2} \\ B_i &= 0, \qquad i \geq 2 \end{aligned}$$

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This gives rise to a particularly simple quantum theory. The kinetic term for $X^{\pm}$ appears as

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The propagator for the $X^{\pm}$ fields is therefore oriented.

$$X^+ \longrightarrow X^-$$

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- The X field has oriented propagators.
- All the interaction vertices in the theory depend only on X^+.
- There are no non-trivial Feynman diagrams in the theory.
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Classically, X^+ is harmonic, and acts as a source for X^-.

Bubble of nothing

By writing the solution to the Laplace equation for X^+ as

$$X^+ = f_+(\sigma^+) + f_-(\sigma^-)$$

the general solution for X^- can be expressed as follows:

$$X^- = g_+(\sigma^+) + g_-(\sigma^-) + \frac{\alpha' \beta \mu^2}{4} \left[\int_{\sigma^+}^{\infty} dy^+ \exp\left(\beta f_+(y^+)\right) \right] \left[\int_{\sigma^-}^{\infty} dy^- \exp\left(\beta f_-(y^-)\right) \right]$$

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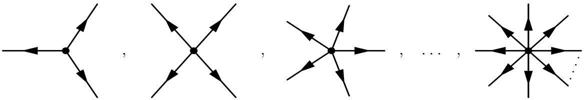
All interaction vertices in the theory depend only on X^+, and therefore correspond to diagrams composed strictly from outgoing lines:



Physical interpretation

The solution can be thought of as a phase boundary in spacetime between the $\mathcal{T} = 0$ phase and the $\mathcal{T} > 0$ phase.

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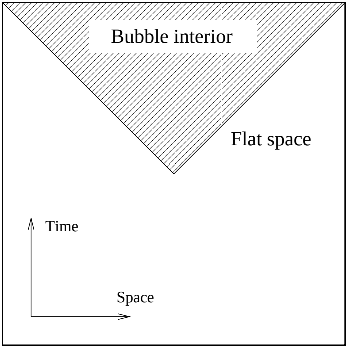
The spacetime picture is therefore a phase bubble expanding out from a nucleation point:



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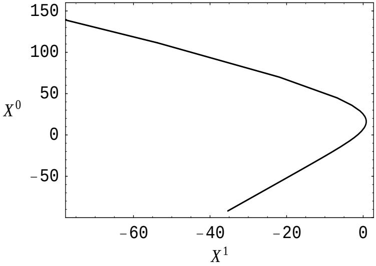
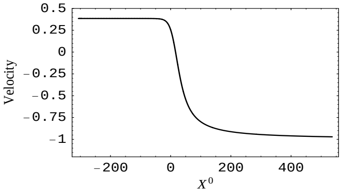


The state collides with the bubble wall and is forced out of the region with nonzero tachyon. (The solution has $\mu^2 = 1,\ \beta = .1,$ and the trajectory corresponds to $p^+ = 3,\ H_\perp \equiv \frac{\alpha' p_i^2}{2} = 4.$)

Physical interpretation

We can also plot the velocity of the particle as a function of time:



So the particle propagates until it hits the bubble wall, where the exponential term becomes important. At that point, the speed of the particle rapidly goes to -1.

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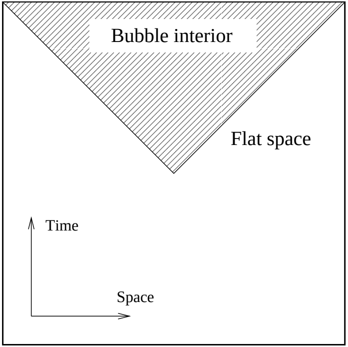

Dimension-changing solutions in the bosonic string

Let's now inroduce some dependence on a third direction:

$$\mathcal{T}(X) = \mu_0^2 \exp\left(\beta X^+\right) - \mu_k^2 \cos(kX_2) \exp\left(\beta_k X^+\right)$$

with

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This is exactly marginal when the wavelength k^{-1} of the tachyon is long compared to the string scale:

$$\mathcal{T}(X^+, X_2) = +\frac{\mu^2}{2\,\alpha'} \exp\left(\beta X^+\right) \;:X_2^2: + \mathcal{T}_0(X^+)$$

$$\mathcal{T}_0(X^+) = \frac{\mu^2\, X^+}{\alpha'\, q\,\sqrt{2}} \exp\left(\beta X^+\right) + \mu'^2 \exp\left(\beta X^+\right)$$

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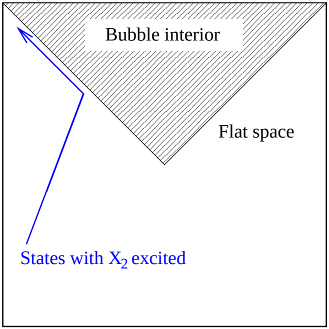
So these string states are pushed out to infinity and disappear from the theory in the late-time limit:



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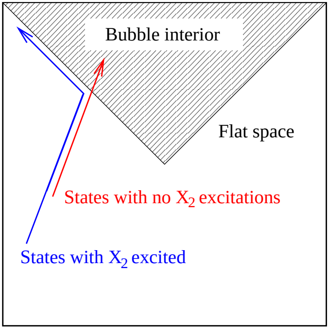

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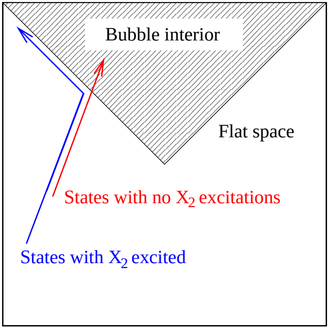

These propagate *through the domain wall and into the bubble region*.

The result is that the amount of matter on the worldsheet decreases dynamically as a function of time.

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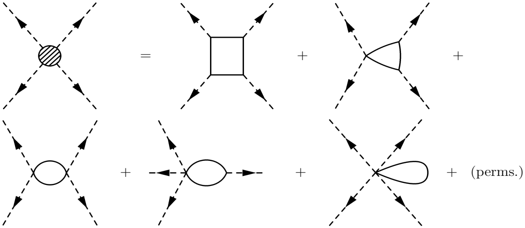
In fact, quantum corrections in this theory truncate at one-loop order:



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In fact: in the far future, all corrections coming from integrating out X_2 decay away, except for three contributions:

- the effective tachyon,
- the dilaton,
- the string-frame metric.

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The remaining contributions are always nonzero, coming from the following diagrams:

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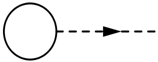

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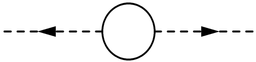

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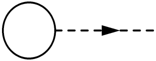
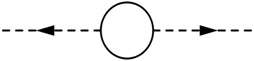
$\Delta(\partial_+\Phi) \ =$ 

$\Delta G_{++} \ =$ 

Write the renormalized dilaton gradient and string-frame metric as:

$$
\begin{aligned}
\hat{V}_\mu &\equiv V_\mu + \Delta V_\mu \\
\hat{G}^{\mu\nu} &\equiv G_{\mu\nu} + \Delta G_{\mu\nu}
\end{aligned}
$$

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We obtain:

$$\hat{V}_- = V_- = -\frac{q}{\sqrt{2}}$$

$$\hat{V}_+ = -\frac{q}{\sqrt{2}} + \frac{\beta}{12}$$

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The result is that the shift in cenral charge contribution from the dilaton precisely cancels the central charge shift due to the reduction in spacetime dimension.

Dimension-changing solutions in the bosonic string

This mechanism of central charge transfer works equally well when the tachyon has a quadratic minimum in several transverse directions:

$$\mathcal{T}(X) = \frac{\mu^2}{2\,\alpha'}\,\exp\left(\beta X^+\right)\,\sum_{i=2}^{n+1} :X_i^2: + \mathcal{T}_0(X^+)$$

$$\mathcal{T}_0(X^+) = \frac{n\,\mu^2\,X^+}{\alpha'\,q\,\sqrt{2}}\exp\left(\beta X^+\right) + \mu'^2\exp\left(\beta X^+\right)$$

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In this case the renormalization of the metric and dilaton leads to a central charge contribution in the $X^+ \to \infty$ limit given by:

$$c^{\mathrm{dilaton}} = 6\alpha'\,\hat{G}^{\mu\nu}\,\hat{V}_\mu\,\hat{V}_\nu = -6\alpha' q^2 + \frac{nq\beta\alpha'}{\sqrt{2}} - \frac{n\alpha'^2 q^2\beta^2}{8} = -(D-26)+n$$

Outline

- Overview of quintessent cosmology and linear dilaton backgrounds
- Supercritical string theory: spacetime effective action
- Stability in time-dependent backgrounds
- Bosonic string solutions with nonzero tachyon
- Dimension-changing solutions in the type 0 string
- Transitions from type 0 to bosonic string theory
- Conclusions

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We will not consider these solutions yet – they are qualitatively different from our previous examples!

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This gives rise to a potential and Yukawa term:

$$\mathcal{L}_{\rm int} = -\frac{\alpha'}{8\pi} G^{MN}\, \partial_M \mathcal{T} \partial_N \mathcal{T} + \frac{i\alpha'}{4\pi} \partial_M \partial_N \mathcal{T}\, \tilde{\psi}^M \psi^N$$

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For our particular choice of profile, this gives an X^+-dependent mass term $M \equiv \mu \exp(\beta X^+)$ to the bosons and fermions in the X_2 and X_3 multiplets:

$$\mathcal{L}_{\rm int} = -\frac{\alpha' \ \mu^2}{8\pi} \exp\left(2\beta X^+\right) \left(\ X_2^2 + X_3^2 \ \right)$$
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The spacetime dimension decreases by 2 in the limit of large X^+! The string theory in D-2 dimensions inherits the diagonal GSO projection of the D-dimensional parent theory – that is, the final-state string theory is still type 0.

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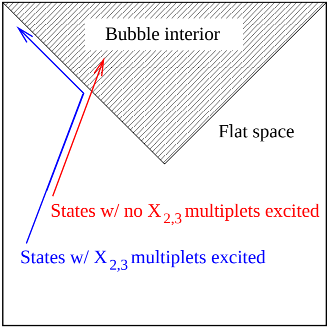

The domain wall is a boundary between type 0 in D dimensions and type 0 in D-2 dimensions.

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The renormalizations are:

$$\Delta\Phi = +\frac{\beta X^+}{2}$$
$$\Delta G_{++} = -\Delta G^{--} = +\frac{\beta^2\alpha'}{2}$$

which gives

$$\Delta c^{dilaton} = +3$$

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In the case $n = K$, the final state is critical, 10-dimensional type 0 string theory with lightlike linear dilaton rolling to weak coupling in the future.

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These symmetries forbid the generation of any term

$$\Delta\mathcal{L} \propto \int d\theta_+ d\theta_- f(X, Y_{n+1,\cdots,K})$$

in the $D-2n$ dimensional theory on the right of the domain wall.

Transitions from type 0 to type II string theory

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The minimum dimension of the final state is therefore **10!**

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This exact solution establishes conclusively that the type 0 theory in supercritical dimensions can relax by tachyon condensation to a supersymmetric ground state in D=10!

Outline

Overview of quintessent cosmology and linear dilaton backgrounds

Supercritical string theory: spacetime effective action

Stability in time-dependent backgrounds

Bosonic string solutions with nonzero tachyon

Dimension-changing solutions in the type 0 string

Transitions from type 0 to bosonic string theory

Conclusions

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We now turn to a related model of lightlike tachyon condensation in type 0 string theory, where the tachyon depends only on X^+, and is independent of the $D-2$ dimensions transverse to $X^\pm$.

We start with the Lagrangian for a timelike linear dilaton theory on a flat worldsheet, describing D free, massless fields and their superpartners:

$$\mathcal{L}_{\rm kin} = \frac{1}{2\pi} G_{MN} \left[\frac{2}{\alpha'}(\partial_+ X^M)(\partial_- X^N) - i\psi^M(\partial_- \psi^N) - i\tilde{\psi}^M(\partial_+ \tilde{\psi}^N) \right]$$

Lightlike tachyon condensation in type 0

The dilaton gradient V_M must satisfy $4\alpha' V^2 = -(D-10)$, so we take

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\begin{aligned}
V_+ &= V_- = -\frac{q}{\sqrt{2}} \\
V_i &= 0, \qquad i = 2, \cdots, D-1 \\
q &\equiv \sqrt{\frac{D-10}{4\alpha'}}
\end{aligned}
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assuming the dilaton rolls to weak coupling in the future.

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$$\mathcal{T} \equiv \tilde{\mu} \exp\left(\beta X^+\right)$$

Lightlike tachyon condensation in type 0

The linearized equation of motion

$$\partial^2 \mathcal{T} - 2V \cdot \partial \mathcal{T} + \frac{2}{\alpha'} \mathcal{T} = 0$$

fixes

$$\beta \boldsymbol{q} = \frac{\sqrt{2}}{\alpha'}$$

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Remember that the tachyon couples to the worldsheet as a $(1,1)$ superpotential, giving rise to a potential and Yukawa term:

$$\mathcal{L}_{\rm int} = -\frac{\alpha'}{8\pi} G^{MN} \, \partial_M \mathcal{T} \partial_N \mathcal{T} + \frac{i\alpha'}{4\pi} \partial_M \partial_N \mathcal{T} \, \tilde{\psi}^M \psi^N$$

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We also get a modified supersymmetry transformation for the fermions:

$$\begin{aligned} \{Q_-, \psi^M\} &= -\{Q_+, \tilde{\psi}^M\} = F^M \\ F^M &\equiv -\sqrt{\frac{\alpha'}{8}}\, G^{MN} \partial_N \mathcal{T} \end{aligned}$$

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is zero.

But there is a nonvanishing F-term and Yukawa coupling between the lightlike fermions:

$$\begin{aligned} F^- &= +\frac{q\sqrt{\alpha'}\mu}{2} \exp\left(\beta X^+\right) \\ \mathcal{L}_{\mathrm{Yukawa}} &= \frac{i\,\mu}{4\pi} \exp\left(\beta X^+\right) \tilde{\psi}^+ \psi^+ \end{aligned}$$

where $\mu \equiv \beta^2 \alpha' \, \tilde{\mu}$.

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The Lagrangian for the light-cone multiplets $X^\mu,\ \psi^\mu,\ \tilde{\psi}^\mu$ is:

$$\begin{aligned}\mathcal{L}_{\rm LC} \;=\; & \frac{i}{\pi}\tilde{\psi}^+\partial_+\tilde{\psi}^- + \frac{i}{\pi}\psi^+\partial_-\psi^- + \frac{iM}{2\pi}\tilde{\psi}^+\psi^+ \\ & -\frac{1}{\pi\alpha'}(\partial_+X^+)(\partial_-X^-) - \frac{1}{\pi\alpha'}(\partial_+X^-)(\partial_-X^+)\end{aligned}$$

where $M \equiv \mu \exp{(\beta X^+)}$.

The $M \to \infty$ limit

The stress tensor of the light-cone sector of the theory is

$$
\begin{aligned}
T^{\mathrm{LC}} &= T^{X^\mu} + T^{\psi^\mu} \\
T^{X^\mu} &\equiv -\frac{1}{\alpha'} G_{\mu\nu} : \partial_+ X^\mu \partial_+ X^\nu : + V_\mu \partial_+^2 X^\mu \\
T^{\psi^\mu} &= +\frac{i}{2} G_{\mu\nu} : \psi^\mu \partial_+ \psi^\nu :
\end{aligned}
$$

with supercurrent

$$
\begin{aligned}
G^{\mathrm{LC}}(\sigma^+) &\equiv \sqrt{\frac{2}{\alpha'}} \psi_\mu (\partial_+ X^\mu) - \sqrt{2\alpha'} V_\mu \partial_+ \psi^\mu \\
&= -\sqrt{\frac{2}{\alpha'}} \psi^+ \partial_+ X^- - \sqrt{\frac{2}{\alpha'}} \psi^- \partial_+ X^+ + \sqrt{\alpha'}\; q\; \partial_+ \psi^+ + \sqrt{\alpha'}\; q\; \partial_+ \psi^-
\end{aligned}
$$

Analogous equations apply for the left-moving stress tensor and supercurrent, replacing ψ with $\tilde{\psi}$ and ∂_+ with ∂_-.

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We would like to define an effective field theory useful for analyzing the large-M regime, described by free effective fields whose interactions are proportional to negative rather than positive powers of M.

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Nothing is integrated out and no information is lost as $M \to \infty$, but the theory becomes free in this limit, when expressed in terms of the new variables.

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We would like to find a new set of variables in which the theory is approximately conformal, with corrections that vanish in the $M_0 \to \infty$ limit:

$$\begin{aligned} \psi^+ &= 2c'_5 - M_0^{-1}\tilde{b}_5 & \qquad \psi^- &= M_0\tilde{c}_5 \\ \tilde{\psi}^+ &= -2\tilde{c}'_5 + M_0^{-1}b_5 & \qquad \tilde{\psi}^- &= -M_0 c_5 \end{aligned}$$

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$$\begin{aligned}\mathcal{L}_{\text{fermi}} &= -\frac{i}{\pi}\tilde{b}_5\partial_+\tilde{c}_5 - \frac{i}{\pi}b_5\partial_- c_5 - \frac{i}{2\pi M_0}b_5\tilde{b}_5 \\ &\quad -\frac{1}{\pi\alpha'}(\partial_+X^+)(\partial_-X^-) - \frac{1}{\pi\alpha'}(\partial_+X^-)(\partial_-X^+)\end{aligned}$$

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Enforcing the equations of motion, the change of variables is

$$\begin{aligned}
\psi^+ &= 2\partial_+ c_5, & \psi^- &= M_0\tilde{c}_5, \\
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The transformation is therefore Lorentz invariant if we assign to b_5 a Lorentz weight of $3/2$, and to c_5 a weight of $-1/2$.

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The central charge of the original $\psi^\pm$ system is 1, but the central charge of a bc ghost system with weights $(3/2, -1/2)$ is -11.

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\psi^+ &= 2c_4' - M^{-1}\tilde{b}_4 + 2\beta(\partial_+ X^+)c_4 \\
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$$

Perform a corresponding redefinition of the bosons $X^\pm$:

$$
\begin{aligned}
X^+ &\equiv Y^+ \\
X^- &\equiv Y^- + i\beta\alpha'\mu\exp\left(\beta X^+\right) c_4\tilde{c}_4
\end{aligned}
$$

Promoting M to a dynamical object

This yields the following Lagrangian

$$
\begin{aligned}
\mathcal{L} \quad = \quad & -\frac{i}{\pi}\tilde{b}_4\partial_+\tilde{c}_4 - \frac{i}{\pi}b_4\partial_- c_4 - \frac{i}{2\pi M}b_4\tilde{b}_4 \\
& -\frac{1}{\pi\alpha'}(\partial_+ Y^+)(\partial_- Y^-) - \frac{1}{\pi\alpha'}(\partial_+ Y^-)(\partial_- Y^+)
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The stress tensor becomes:

$$T^{Y^\mu} + T^{\psi^\mu} = -\frac{1}{\alpha'}G_{\mu\nu}\partial_+ Y^\mu \partial_+ Y^\nu + V_\mu \partial^2 Y^\mu - \frac{3i}{2}\partial_+ c_4\, b_4 - \frac{i}{2}c_4\,\partial_+ b_4$$

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As M grows, the stress tensor becomes free in canonical variables, with all interaction terms going to zero as M^{-1}.

Promoting M to a dynamical object

We refer to the variables $Y^\mu,\ b_4,\ c_4,\ \tilde{b}_4,\ \tilde{c}_4$ as the *IR variables*, and the $X^\mu,\ \psi^\mu,\ \tilde{\psi}^\mu$ as the *UV variables*.

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There is an exact *duality* between the UV description and the IR description.

In the case at hand, loop corrections are trivial on both sides, and the duality inverts the expansion parameter for *conformal perturbation theory* rather than for the loop expansion.

Promoting M to a dynamical object

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Question: How does this work?

Renormalization of the dilaton gradient

It turns out that the natural normal-ordering prescription for the UV variables agrees only up to finite terms with the natural orderings for composite operators in the IR variables.

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It turns out that the natural normal-ordering prescription for the UV variables agrees only up to finite terms with the natural orderings for composite operators in the IR variables.

The effect of these finite differences will be to renormalize the dilaton gradient of the system by an amount $\Delta V_+ = \beta,\ \Delta V_- = 0$.

Normal ordering in the UV variables

Using the properties of Feynman diagrams and the equations of motion, we can derive modified OPEs for the UV fields.

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Using the properties of Feynman diagrams and the equations of motion, we can derive modified OPEs for the UV fields.

The natural basis for operators in the UV description is a basis of normal-ordered products

$$: X^{\mu_1}(\rho_1)\cdots X^{\mu_m}(\rho_m)\,\psi^{\nu_1}(\sigma_1)\cdots\psi^{\nu_n}(\sigma_n)\tilde{\psi}^{\pi_1}(\tau_1)\cdots\tilde{\psi}^{\pi_p}(\tau_p) :$$

- The normal-ordered operator is nonsingular when any of the arguments in the normal ordering symbol approach one another;
- The normal-ordered operators obey the equations of motion. For instance:

$$\partial_{\tau^+}\partial_{\tau^-} : X^-(\sigma)X^-(\tau) : \; = -\frac{i\beta\alpha'\mu}{4} : X^-(\sigma)\exp\left(\beta X^+(\tau)\right)\,\tilde{\psi}^+(\tau)\psi^+(\tau) :$$

- The normal ordered product of two "$+$" operators is equal to the ordinary product;

Normal ordering in the UV variables

- The normal ordered product of a "$+$" field and a "$-$" field is defined with the subtraction prescription of the free theory;
- The normal ordered product of two "$-$" fields has only "$+$" fields on the right-hand side, and scales as a single power of M;
- In the limit $M \to 0$, the structure of the algebra of the operators becomes that of the free theory (this property is implied by the three previous properties).

Given these properties, we can derive the full structure of the OPE for UV fields.

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Define a second normal ordering prescription, appropriate to the IR limit of the theory. In this case we take our basis of operators to be

$$\substack{\circ \\ \circ} Y^{\mu_1}(\rho_1)\cdots Y^{\mu_m}(\rho_m)\, b_4(\sigma_1)\cdots b_4(\sigma_n)\tilde{b}_4(\tau_1)\cdots \tilde{b}_4(\tau_p)c_4(\zeta_1)\cdots c_4(\zeta_q)\tilde{c}_4(\omega_1)\cdots \tilde{c}_4(\omega_r) \substack{\circ \\ \circ},$$

- The normal-ordered operator is nonsingular when any of the arguments of operators in the normal ordering symbol approach one another;
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$$\partial_{\tau^+}\partial_{\tau^-} \substack{\circ \\ \circ} Y^-(\sigma)Y^-(\tau) \substack{\circ \\ \circ} = -\frac{i\beta\alpha'}{4\mu} \substack{\circ \\ \circ} Y^-(\sigma)\exp\left(-\beta Y^+(\tau)\right)\, b_4(\tau)\tilde{b}_4(\tau) \substack{\circ \\ \circ}\ ;$$

Normal ordering in the IR variables

- The normal ordered product of two operators from the set $b_4, \tilde{b}_4, Y^+$ is equal to the ordinary product;
- The normal ordered product of a field from the set $c_4, \tilde{c}_4, Y^-$ with a field from the set $b_4, \tilde{b}_4, Y^+$ is defined with the subtraction prescription of the free theory;
- The normal ordered product of two fields from the set $c_4, \tilde{c}_4, Y^-$ has only fields from the set $b_4, \tilde{b}_4, Y^+$ on the right-hand side, and scales as a single power of M^{-1};
- In the limit $M \to \infty$, the structure of the algebra of the operators becomes that of the free theory of the IR fields.

Normal ordering in the IR variables

The bosonic stress tensor turns out to transform unproblematically, but the fermionic stress tensor picks up a quantum correction due to the mismatch between $:\;\;:$ and $\circ\atop\circ$ $\circ\atop\circ$ normal ordering prescriptions.

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We are left with a contribution to the central charge equal to

$$\begin{aligned} c^{\text{dilaton}} &= 6\alpha' \eta^{\mu\nu} \hat{V}_\mu \hat{V}_\nu = -6\alpha' q^2 + 6\sqrt{2}\alpha' \beta q \\ &= 27 - \frac{3D}{2} \end{aligned}$$

Quantum corrections

We have the remaining central charge contributions

- $+2$ from the Y^μ
- -11 from the $b_4 c_4$ system
- $\frac{3}{2}(D-2)$ from the transverse degrees of freedom $X^i,\ \psi^i$
- total free-field contribution of $\frac{3D}{2} - 12$

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The central charge being transferred to the dilaton gradient does *not* occur through a loop diagram of massive fields being integrated out.

Instead, the central charge is transferred through a mismatch of normal-ordering prescriptions appropriate to the free field theories in the two limits $X^+ \to \pm\infty$.

Quantum corrections

Break up the supercurrent: $G^{\mathrm{LC}} = \mathbf{1} + \mathbf{2} + \mathbf{3} + \mathbf{4}$, with

$$\mathbf{1} \equiv -\sqrt{\frac{2}{\alpha'}}\psi^{+}(\partial_{+}X^{-}) \qquad \mathbf{2} \equiv -\sqrt{\frac{2}{\alpha'}}\psi^{-}(\partial_{+}X^{+})$$

$$\mathbf{3} \equiv \sqrt{\alpha'}\, q\, \partial_{+}\psi^{+} \qquad \mathbf{4} \equiv \sqrt{\alpha'}\, q\, \partial_{+}\psi^{-}$$

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$$\mathbf{3} \equiv \sqrt{\alpha'}\, q\, \partial_{+}\psi^{+} \qquad \mathbf{4} \equiv \sqrt{\alpha'}\, q\, \partial_{+}\psi^{-}$$

The full transformation of the supercurrent from UV to IR variables is

$$\begin{aligned}
\mathbf{1}_{\text{classical}} &= -2\sqrt{\frac{2}{\alpha'}} \,{}^{\circ}_{\circ} \left[(\partial_{+}c_4)(\partial_{+}Y^{-}) + \beta c_4(\partial_{+}Y^{+})(\partial_{+}Y^{-}) - \frac{i\beta\alpha'}{2}(\partial_{+}c_4)b_4 c_4 \right] {}^{\circ}_{\circ} \\
\mathbf{1}_{\text{quantum}} &= -\beta\sqrt{\frac{\alpha'}{2}}\partial_{+}^{2}c_4 - 2\beta^{2}\sqrt{\frac{\alpha'}{2}}c_4\partial_{+}^{2}Y^{+} \\
\mathbf{2}+\mathbf{4} &= \frac{q}{2}\sqrt{\alpha'}\,b_4 \\
\mathbf{3} &= 2q\sqrt{\alpha'}(\partial_{+}^{2}c_4) + 2\sqrt{\frac{2}{\alpha'}}(\partial_{+}Y^{+})(\partial_{+}c_4) + 2\sqrt{\frac{2}{\alpha'}}c_4(\partial_{+}^{2}Y^{+})
\end{aligned}$$

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Expressed in $b_4, \ c_4, \ Y$ variables, the supercurrent is manifestly finite in the limit $X^+ \to +\infty$ (as is the stress tensor).

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$$\boxed{\text{The } b_4,\ c_4,\ Y \text{ fields can indeed be regarded as dual variables that render the theory free in the } M \to \infty \text{ limit.}}$$

The IR limit

We now focus strictly on the limiting regime of the IR theory.

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In practice, this means that, when written in IR variables, we discard the $\exp(-\beta Y^+)\,\tilde{b}_4 b_4$ term in the action, as well as any $\exp(-\beta Y^+)$ terms in the supercurrent and stress tensor.

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Rescale the b_4 field so that the new b fermion appears in the supercurrent with unit normalization. To preserve all canonical commutators, however, we will rescale the c_4 field oppositely:

$$b_4 = \frac{2}{q\sqrt{\alpha'}} b_3 = \beta\sqrt{2\alpha'}\, b_3$$

$$c_4 = \frac{q\sqrt{\alpha'}}{2} c_3 = \frac{1}{\beta\sqrt{2\alpha'}} c_3$$

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We would like to find field variables that render this discrete symmetry more manifest, such that only the vector $\hat{V}_\mu$ enters G^{LC}. We therefore define new variables $b_2,\ c_2,\ Z^\mu$ by:

$$
\begin{aligned}
Y^\pm &= Z^\pm \pm \frac{i}{2\beta} c_2 \partial_+ c_2 \\
b_3 &= b_2 - \frac{2}{\beta\alpha'}(\partial_+ c_2)\left(\ \partial_+ Z^+ - \partial_+ Z^-\ \right) - \frac{1}{\beta\alpha'} c_2 \left(\ \partial_+^2 Z^+ - \partial_+^2 Z^-\ \right) \\
&\quad + \frac{i}{2\beta^2\alpha'} c_2 (\partial_+ c_2)(\partial_+^2 c_2) \\
c_3 &= c_2
\end{aligned}
$$

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The bosons Z^μ transform into their own derivatives, times a goldstone fermion:

$$[Q, Z^\mu] = i c_2 \partial_+ Z^\mu \qquad \{Q, c_2\} = 1 + i c_2 \partial_+ c_2$$

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$$Q \equiv \frac{1}{2\pi} \int d\sigma_1 \, G(\sigma)$$

In the sector involving the transverse fields X_i, ψ^i, supersymmetry is realized in the usual linear fashion:

$$\begin{aligned} [Q, X_i] &= i\sqrt{\frac{\alpha'}{2}}\psi^i \\ \{Q, \psi^i\} &= \sqrt{\frac{2}{\alpha'}}\partial_+ X_i \end{aligned}$$

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However, it turns out that this realization is equivalent to one for which worldsheet supersymmetry is realized completely nonlinearly in *all* sectors.

We now perform a final transformation on the system. Defining the Hermitian infinitesimal generator

$$g \equiv -\frac{i}{2\pi} \int d\sigma_1 c_2(\sigma) G^{\perp}(\sigma)$$

we transform all operators in the theory according to

$$\mathcal{O} \to U\,\mathcal{O}\,U^{-1}$$

with

$$U \equiv \exp\left(ig\right)$$

The IR limit

The total final supercurrent $G \equiv G^{\mathrm{LC}} + G^{\perp}$ is then

$$
\begin{aligned}
G &= b_1 + ic_1' b_1 c_1 - c_1 T^{\mathrm{mat}} + c_1'' \left(-\frac{1}{6} c^{\perp} - \frac{1}{2} + \alpha' q^2 \right) \\
&\quad + c_1 c_1' c_1'' \left(-\frac{i}{4} \alpha' q^2 - \frac{i}{2} + \frac{i}{24} c^{\perp} \right)
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And the total transformed stress tensor is

$$T = T^{\mathrm{mat}} + T^{b_1 c_1}$$

with

$$T^{b_1 c_1} = -\frac{3i}{2} \partial_+ c_1 b_1 - \frac{i}{2} c_1 \partial_+ b_1 + \frac{i}{2} \partial_+ (c_1 \partial_+^2 c_1)$$

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Plugging in $q = \sqrt{\frac{D-10}{4\alpha'}}$ and $c^{\perp} = \frac{3}{2}(D-2)$:

$$G = b_1 + ic_1' b_1 c_1 - c_1 T^{\mathrm{mat}} - \frac{5}{2} c_1''$$

The IR limit

The $X^+ \to \infty$ limit of our solution is described by a free worldsheet theory with a bc ghost system of weights $(3/2, -1/2)$, D free scalars Z^M and $D-2$ free fermions ψ^{Z^i}.

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The theory has critical central charge for a SCFT interpreted as the worldsheet theory of a RNS superstring in conformal gauge.

Berkovits-Vafa construction

This type of superconformal field theory belongs to a class of constructions introduced by Berkovits and Vafa, in which the bosonic string is embedded in the solution space of the superstring.

[hep-th/9310170]

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For a conformal field theory T^{mat} with a central charge of 26, it is possible to construct a corresponding superconformal field theory defined by $G,\ T$ with central charge 15.

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For a conformal field theory T^{mat} with a central charge of 26, it is possible to construct a corresponding superconformal field theory defined by $G, \ T$ with central charge 15.

Upon treating the superconformal theory as a superstring theory, the resulting physical states and scattering amplitudes are identical to those of the theory defined by T^{mat} when treated as a bosonic string theory.

Berkovits-Vafa construction

The construction can be summarized as follows:

- Given a conformal stress tensor T^{mat} with central charge 26, a ghost system $b_1 c_1$ can be introduced with weights $(3/2, -1/2)$ and stress tensor $T^{b_1 c_1}$.

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- This gives rise to a fermionic primary current of weight $3/2$:

$$G \equiv b_1 + i c_1' b_1 c_1 - c_1 T^{\mathrm{mat}} - \frac{5}{2} c_1''$$

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$$G \equiv b_1 + i c_1' b_1 c_1 - c_1 T^{\mathrm{mat}} - \frac{5}{2} c_1''$$

- This closes on the stress tensor of the theory:

$$G(\sigma) G(\tau) \simeq \frac{10i}{(\tau^+ - \sigma^+)^3} + \frac{2i}{(\tau^+ - \sigma^+)} T^{\mathrm{total}}(\tau)$$

where

$$T^{\mathrm{total}} \equiv T^{\mathrm{mat}} + T^{b_1 c_1}$$

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$$\begin{aligned} L_n^{\mathrm{mat}}\,|\mathcal{U}\rangle &= 0 \qquad\qquad n \geq 1 \\ L_0^{\mathrm{mat}}\,|\mathcal{U}\rangle &= 1 \end{aligned}$$

We have an exact solution describing a dynamical transition between string theories that differ from one another in their worldsheet gauge algebra.

Transition to bosonic string theory

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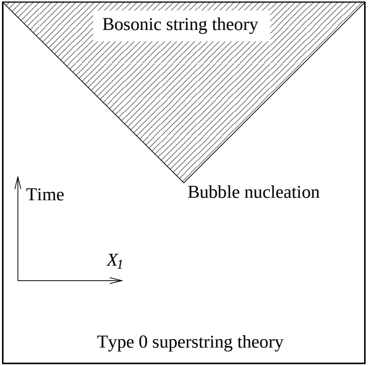
The dynamics then spontaneously break worldsheet supersymmetry, giving rise to a bosonic string theory in the same number of dimensions deep inside the tachyonic phase.

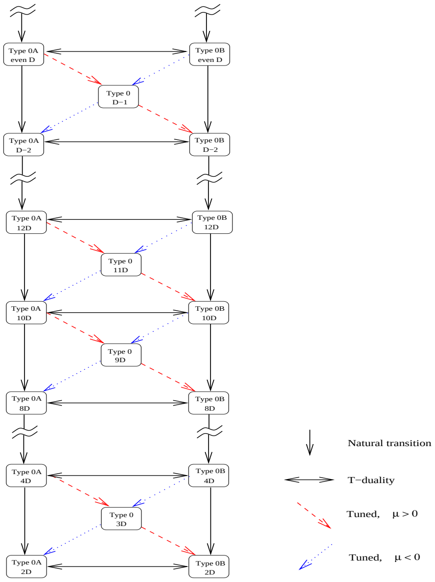


Outline

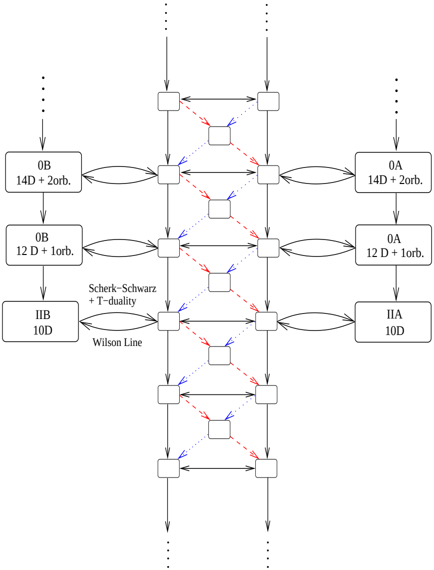
A partial catalog of exact transitions

start	D_{init}	$\exp\left(-\beta X^{+}\right)\mathcal{T}$	end	D_{fin}	comments
bos	D	$\mu^2\ X_2^2$	bos	D-1	tuned
0	D	$\mu X_2 X_3$	0	D-2	natural
0 (orb)	D	$\mu X_{i+1} Y_i$	II	10	stable
0	D	μ	bos	D $+\frac{1}{2}$ (D-2)	tuned
UHE	10	μX_2	HE9	9	stable
$\mathrm{HO}^{(+1)}$	11	μX_2	HO	10	stable
$\mathrm{HO}^{(+1)/}$	11	μX_2	$\mathrm{HO}^{/}$	10	natural
$\mathrm{HO}^{(+1)/}$ (orb)	11	μX_2	HO	10	stable
$\mathcal{N} = 2$	2 D_c - 1	$\mu\phi_2\phi_3$	$\mathcal{N} = 2$	2 D_c - 5	natural
$\mathcal{N} = 2$	2 D_c - 1	μ	*bos*	3 D_c - 2	tuned

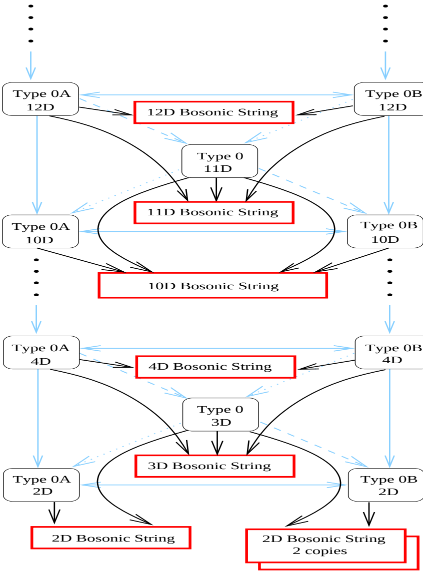
The Big Picture – Part I



The Big Picture – Part II



The Big Picture – Part III



Conclusions

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- Thank you!