Modified gravity as an alternative to dark energy

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Cosmic acceleration

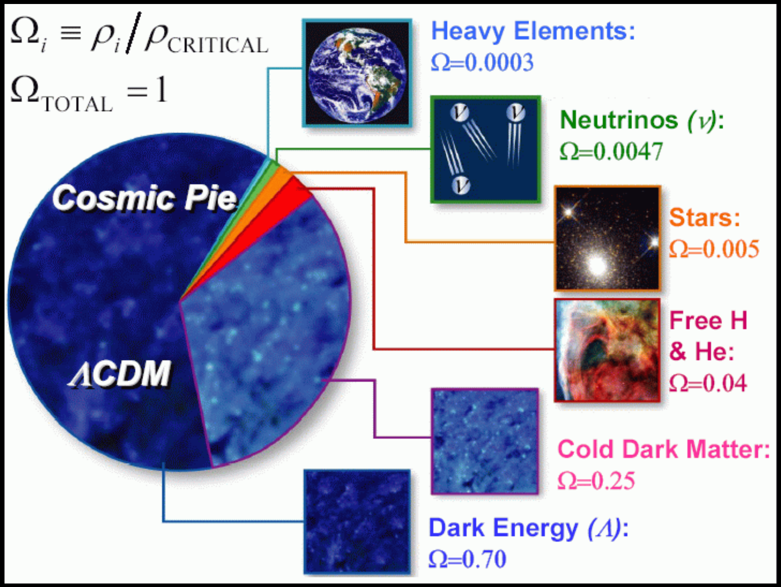
- Cosmic acceleration

  Big surprise in cosmology

- Simplest best fit model

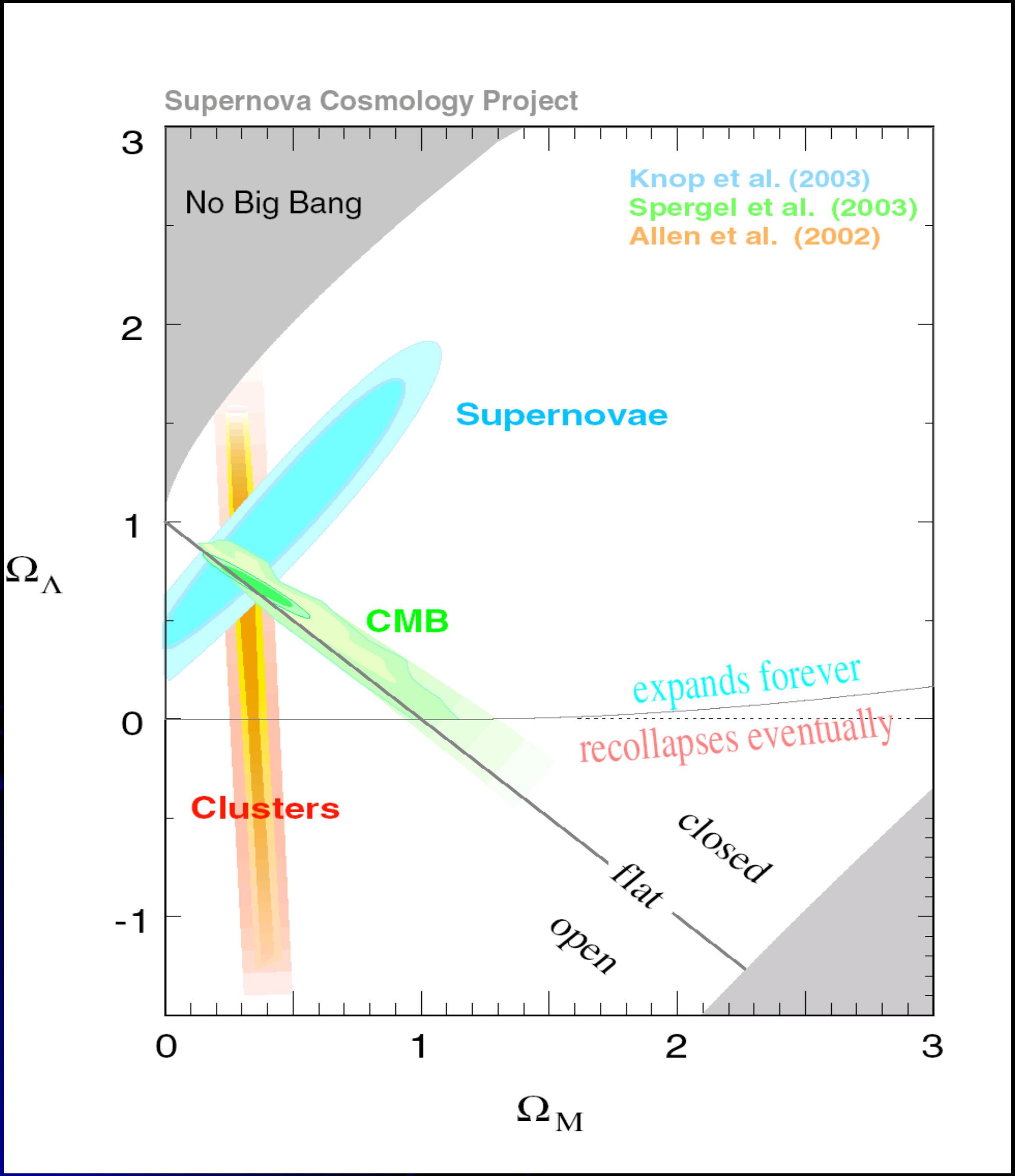
  LCDM

  4D general relativity + cosmological const.

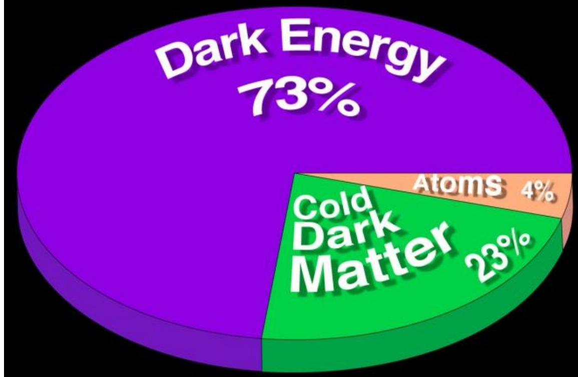


$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3} + \frac{K}{a^2}$$

$$\Rightarrow \quad 1 = \Omega_m + \Omega_\Lambda + \Omega_K$$



- 3 independent data sets intersect



Problem of LCDM

- Huge difference in scales (theory vs observation)

$$\rho_\Lambda\Big|_{obs} = \frac{\Lambda}{8\pi G} \sim H_0^2 M_{pl}^2 \sim (10^{-33}\,\mathrm{eV})^2 (10^{19}\,\mathrm{GeV})^2 \sim 10^{-47}\,\mathrm{GeV}^4$$

$$\rho_\Lambda\Big|_{theory} \sim M^4_{\mathrm{fundamental}} > (10^3\,\mathrm{GeV})^4 \sim 10^{12}\,\mathrm{GeV}^4$$

vacuum energy =0 from fundamental theory

(1) tiny vacuum energy is left somehow

(2) potential energy of quintessence field

Alternative models

- Tiny energy scale

  unstable under quantum corrections

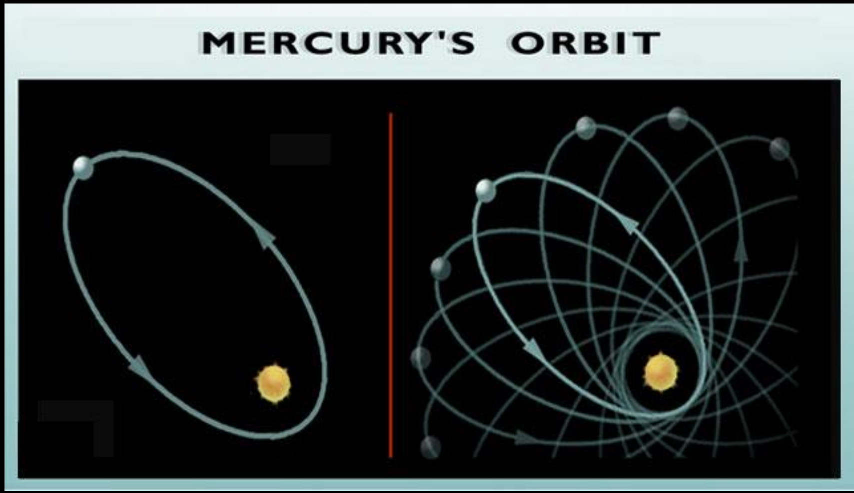
- Alternative - modified gravity

  dark energy is important only at late times

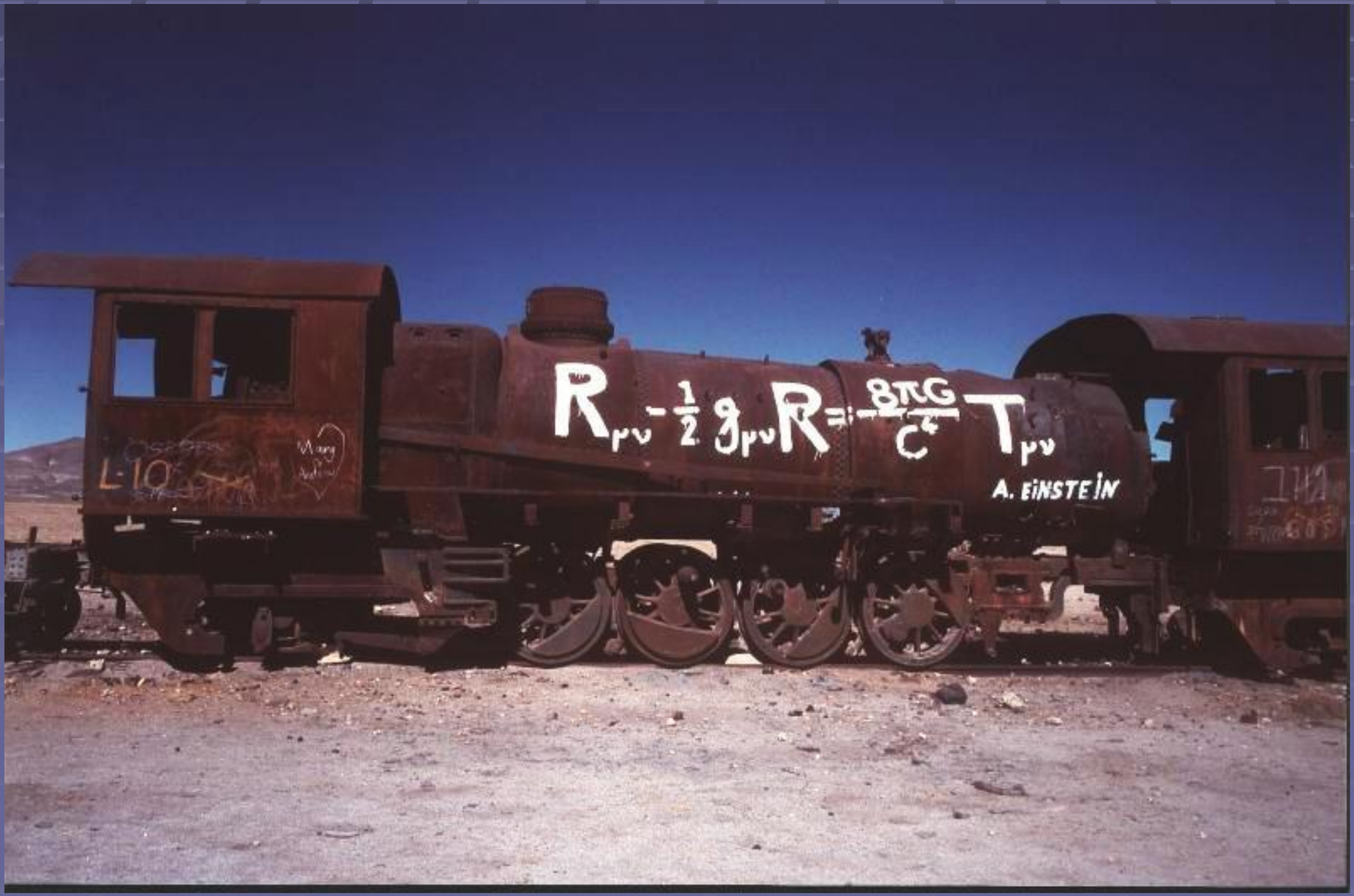
  ➡ large scales / low energy modifications

cf. precession of perihelion

dark planet v GR



Is cosmology probing breakdown of GR on large (IR) scales ?



Problems of IR modification

- Modified gravity

  graviton has a scalar mode

  Solar system constraints - theory must be GR

$$S = \int d^4x\sqrt{-g}\left(\Psi R + \frac{\omega(\Psi)}{\Psi}(\nabla\Psi)^2 + V(\Psi)\right)$$

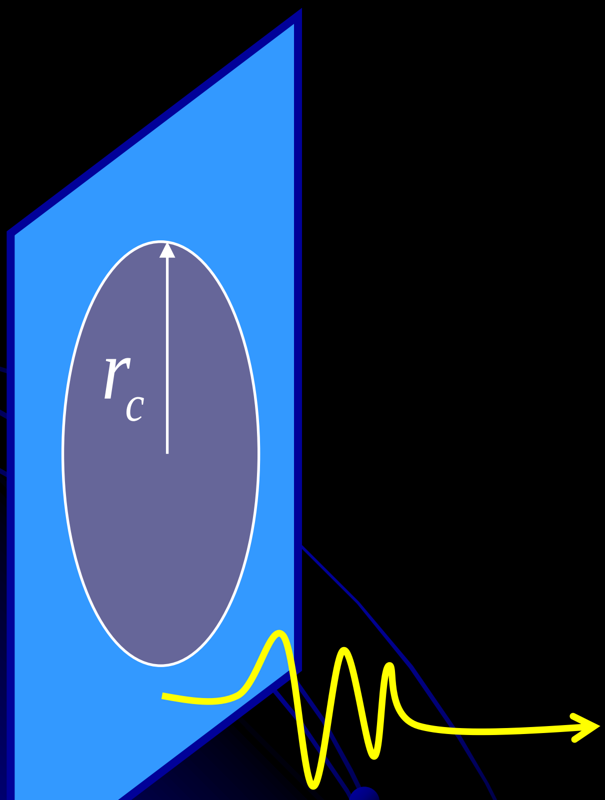
$$\omega > 10000, \quad (V \sim H_0^2 M_{pl}^2)$$

cf. $f(R) = R + \dfrac{\mu}{R} \quad \Longrightarrow \quad \omega = 0$ (Chiba)

difficult to explain dark energy purely from modified gravity

DGP model (Dvali, Gabadadze,Porrati)

$$S=\frac{1}{32\pi G\, r_c}\int d^5x\sqrt{-{}^{(5)}g}\;{}^{(5)}R+\frac{1}{16\pi G}\int d^4x\sqrt{-g}\left(R+L_m\right)$$



gravity leakage

Infinite extra-dimension

- Crossover scale r_c

$r < r_c$ 4D Newtonian gravity

$r > r_c$ 5D Newtonian gravity

Consistent with local experiments?

- DGP also has a scalar mode of graviton

  $r < r_c$:4D Newtonian but not 4D GR!

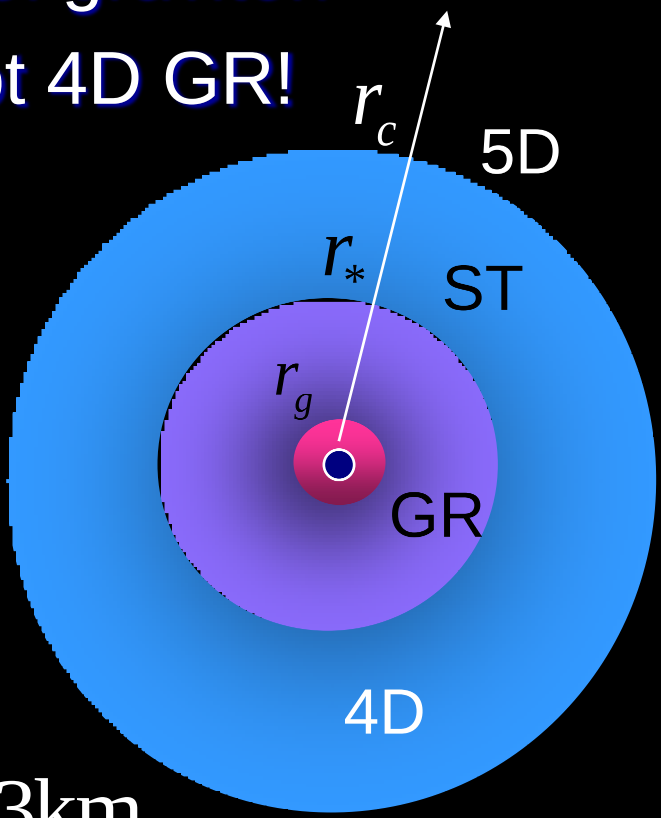
  (Scalar-Tensor theory)

- Non-linear shielding

  theory becomes GR at

$$r < r_* = \left(r_g r_c^2\right)^{\frac{1}{3}}$$

(Deffayet et.al.)



solar-system $r_g = 2GM_{\odot} \approx 3\text{km}$

constraints can be evaded if $r_c > H_0^{-1} \approx 10^{28}\,\text{cm}$

Based on DGP model, we will see how we can distinguish between modified gravity models from LCDM and dark energy models in GR

Cosmology of DGP

$$\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R + \int d^4x\, L_m$$

$$\frac{1}{32\pi G\, r_c}\int d^5x\sqrt{-{}^{(5)}g}\;{}^{(5)}R$$

- Friedmann equation (Deffayet)

$$\frac{H}{r_c} = H^2 - \frac{8\pi G}{3}\rho$$

early times $Hr_c \to \infty$ 4D Friedmann

late times $\rho \to 0$ $\quad H \to \frac{1}{r_c}$

As simple as LCDM model

and as fine-tuned as LCDM $r_c \sim H_0^{-1}$

(stability against quantum corrections can be different)

LCDM vs DGP

- Can we distinguish between DGP and LCDM ?

  Friedmann equation

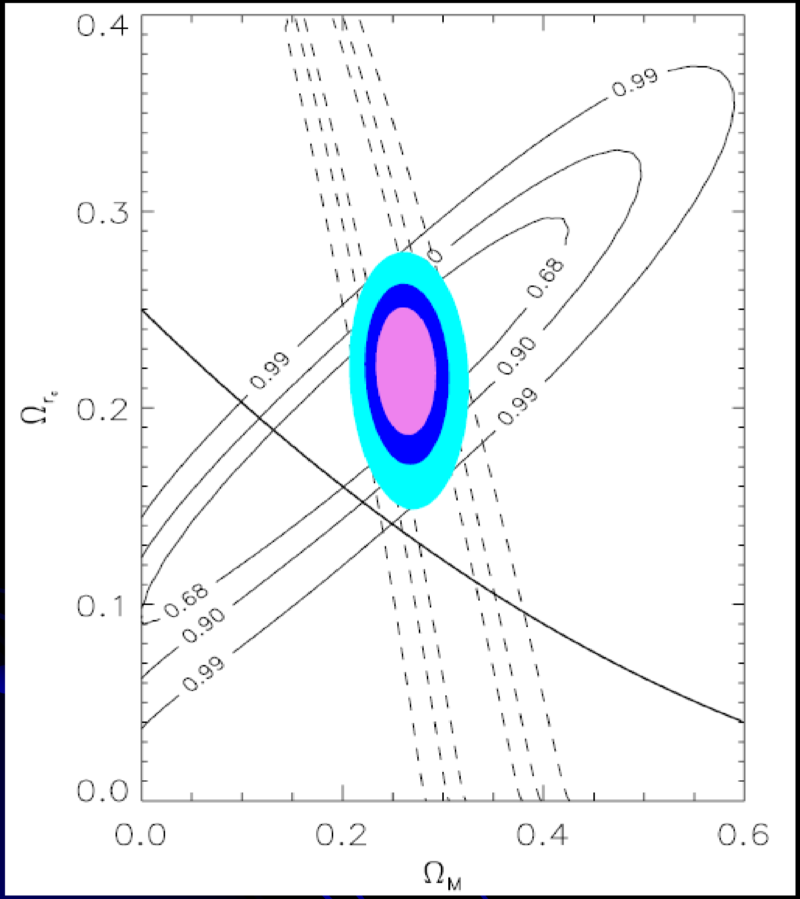
$$H^2 = \left(\frac{1}{2r_c} + \sqrt{\frac{1}{4r_c^2} + \frac{8\pi G}{3}\rho_m} \right)^2 + \frac{K}{a^2}$$

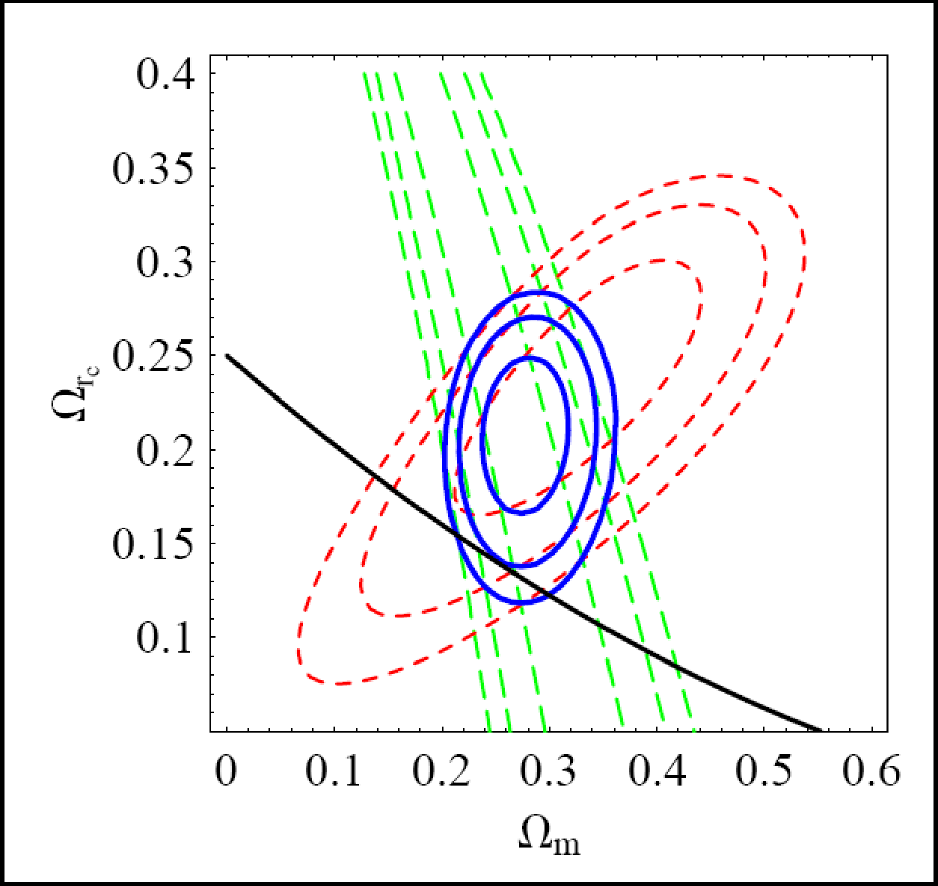
$$\Rightarrow \quad 1 = \left(\sqrt{\Omega_{r_c}} + \sqrt{\Omega_{r_c} + \Omega_m} \right)^2 + \Omega_K, \quad \Omega_{r_c} = \frac{1}{4r_c^2 H_0^2}$$

cf. LCDM

$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{\Lambda}{3} + \frac{K}{a^2} \quad \Rightarrow \quad 1 = \Omega_m + \Omega_\Lambda + \Omega_K$$

SNe + baryon oscillation





SNLS + SDSS

(Fairbairn and Goobar astro-ph/0511029)

'Gold' set + SDSS

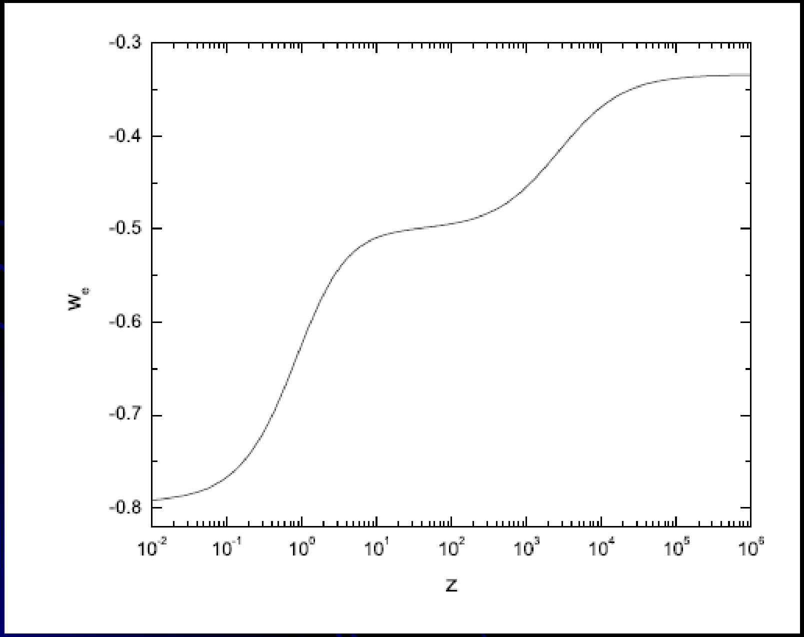
(Maartens and Majerotto astro-ph/0603353)

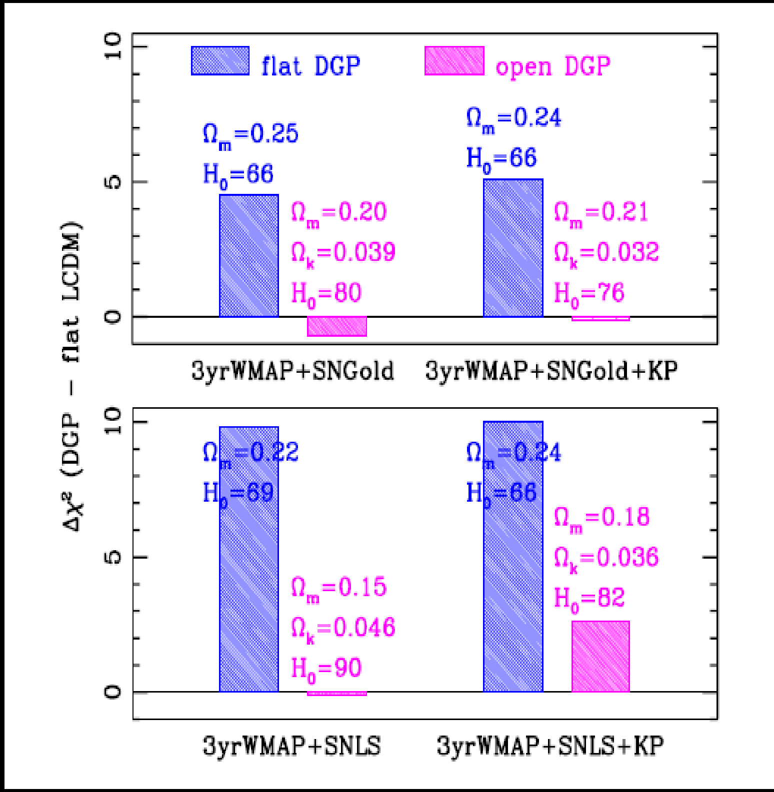
(cf. Alam and Sahni, astro-ph/0511473)

- flat model conflicts with data

$$H^2 = \frac{8\pi G}{3}\rho_m + \frac{H}{r_c}$$

$$\rho_{de} = \frac{H}{r_c}, \quad w_{de} = -\frac{1}{1+\Omega_m(a)}$$





(Song, Hu and Sawicki)

inclusion of curvature (open universe) improves a fit

DGP Cosmology

- As simple as LCDM

  a falsifiable model

  now the model is under pressure from the data

  flat model

  measurements of Ω_m is crucial $w_{de} = -\frac{1}{1+\Omega_m(a)}$

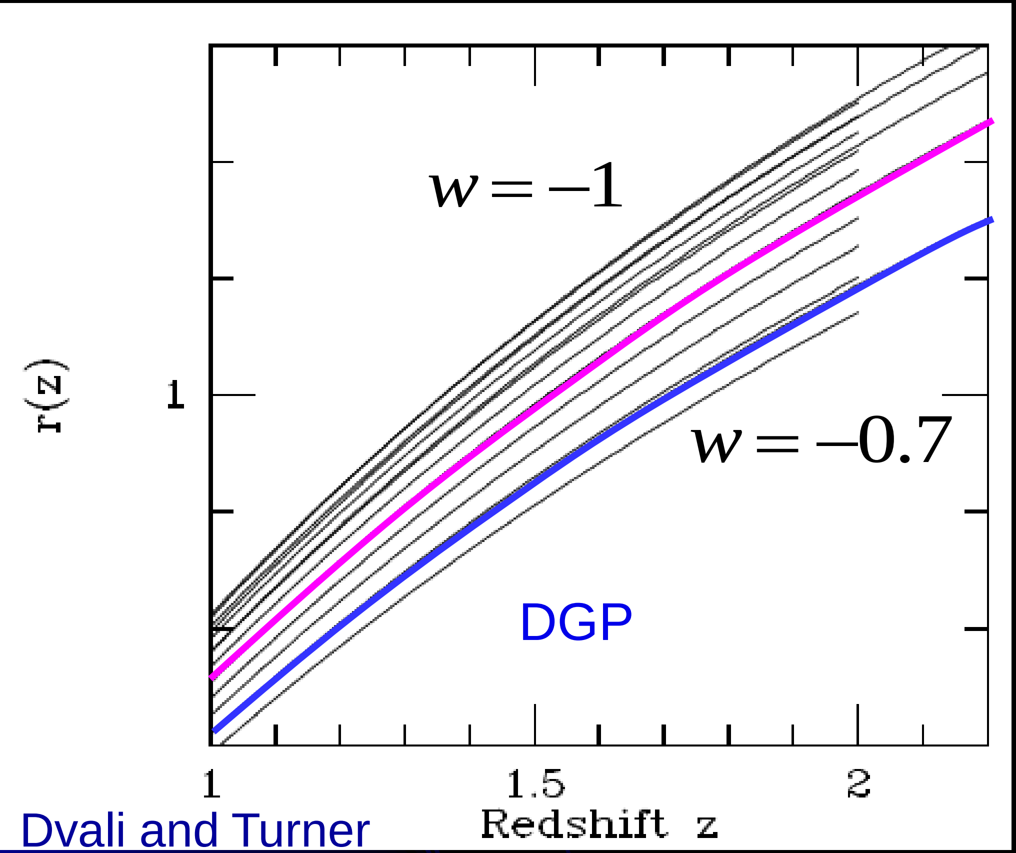
- Fit to SNe assuming flat universe

$$r_c \approx 1.4H_0^{-1}$$

A parameter is fixed!

Dark energy vs DGP

- Can we distinguish between dark energy in GR and DGP ?



$$r(z) = \int_0^z dz\, H(z)^{-1}$$

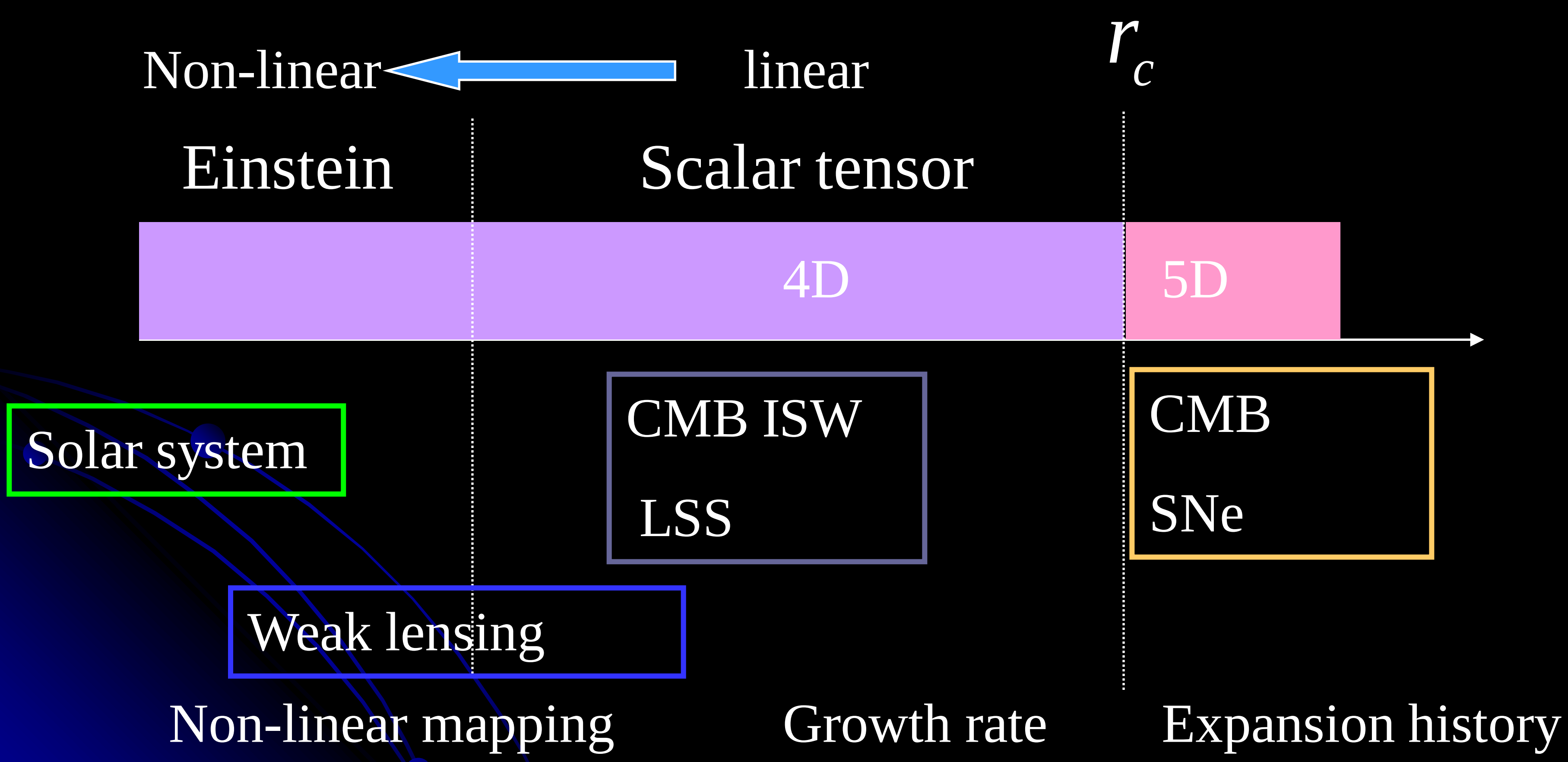
DGP model is fitted by

$$w(a) = w_0 + w_a(1-a),$$

$$w_0 = -0.78,\ w_a = 0.32$$

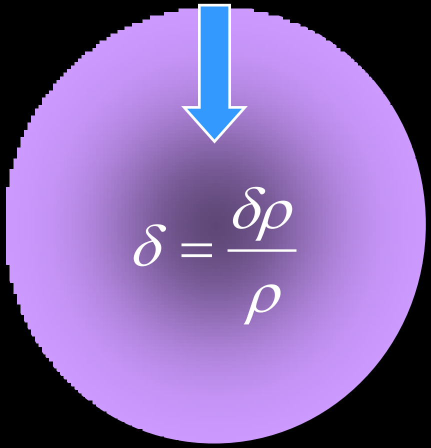
(Linder)

Cosmology as a probe of DGP gravity



Growth rate of structure formation

- Evolution of CDM over-density



GR

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_4 \rho\delta$$

If there is no dark energy $\delta \propto a$

dark energy suppresses the gravitational collapse

DGP

$$\ddot{\delta} + 2H\dot{\delta} = 4\pi G_4 F(Hr_c)\rho\delta$$

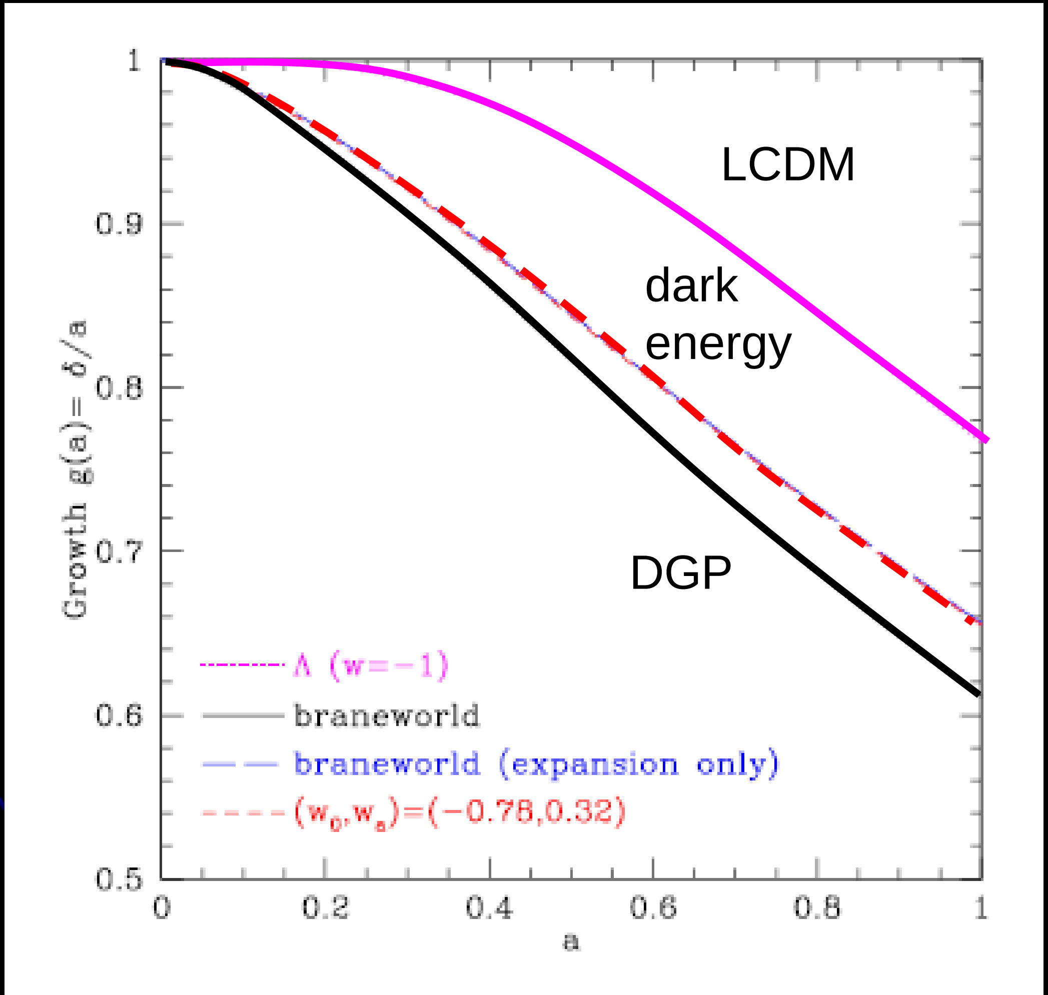
an additional modification from the scalar mode

Expansion history vs growth rate

(Lue.et.al, Linder, KK & Maartens, KK)

- Growth rate resolves the degeneracy

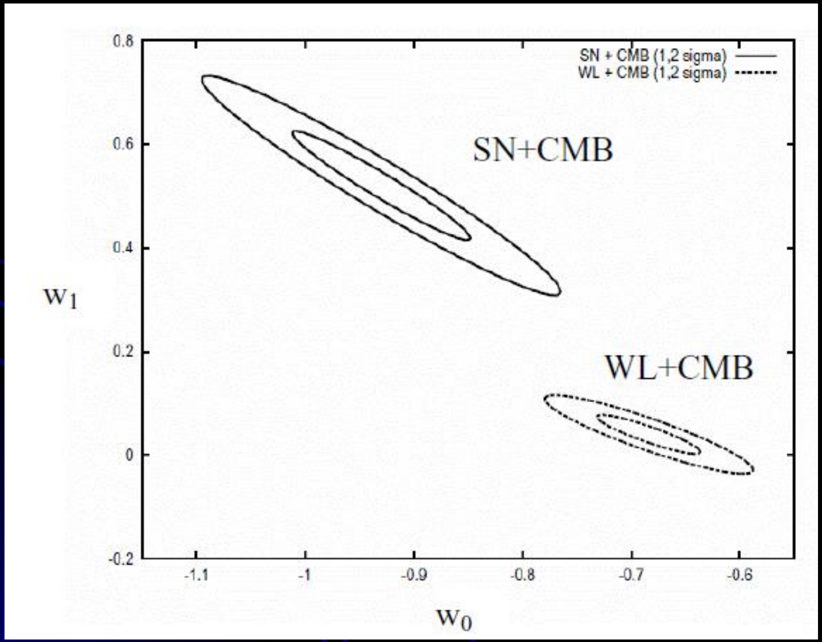
$$g(a) = \frac{\delta}{a}$$



Experiments

(Ishak, Upadhye, Spergel astro-ph/0507184)

- ASSUME our universe is DGP braneworld
  but you do not want to believe this,
  so fit the data using dark energy model



$$w(z) = w_0 + w_1 z,$$

m(z):
apparent magnitude

R:
CMB shift parameter

G(a):
Growth rate

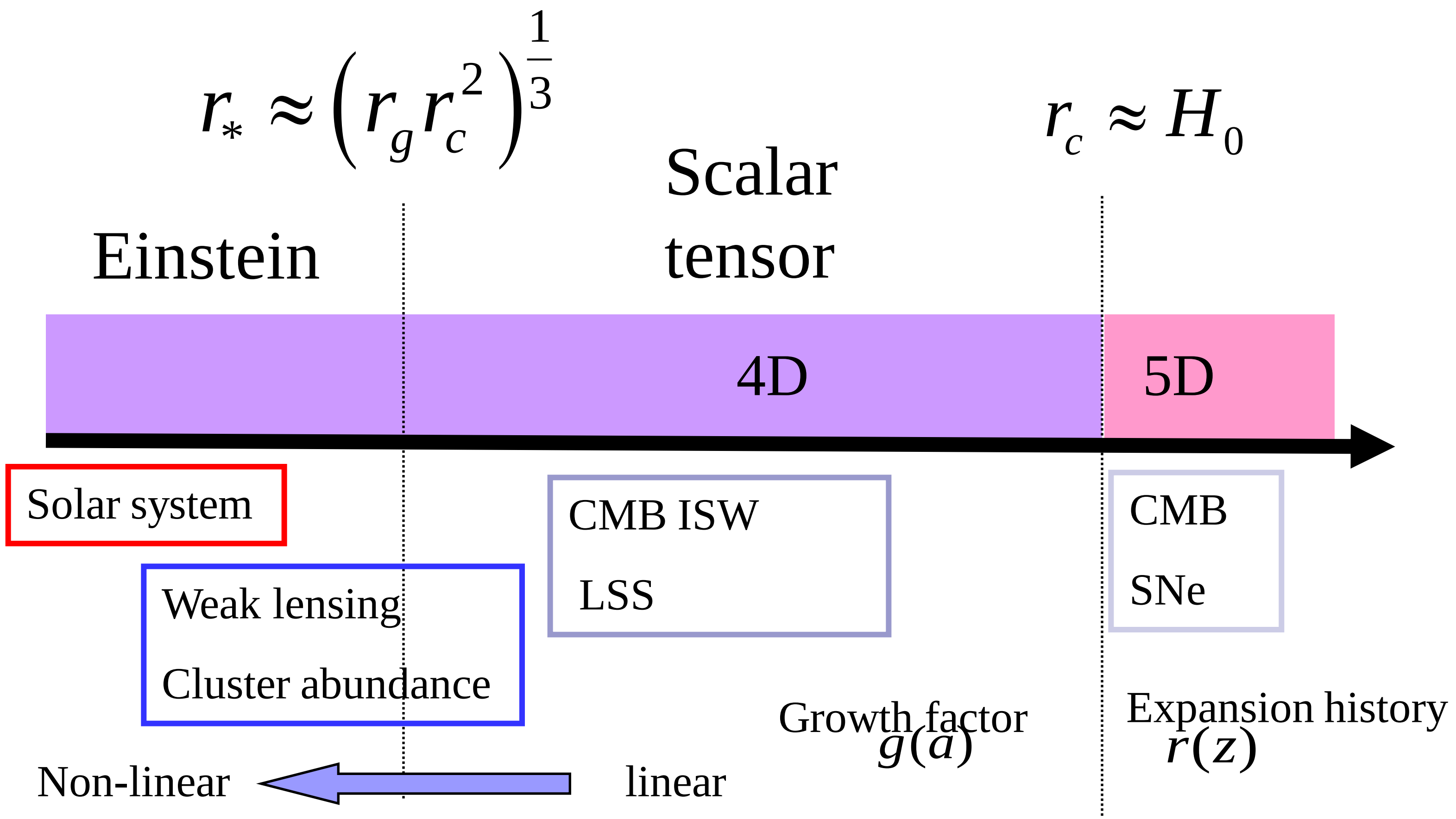
OR

SNe+CMB

SNe+weak lensing

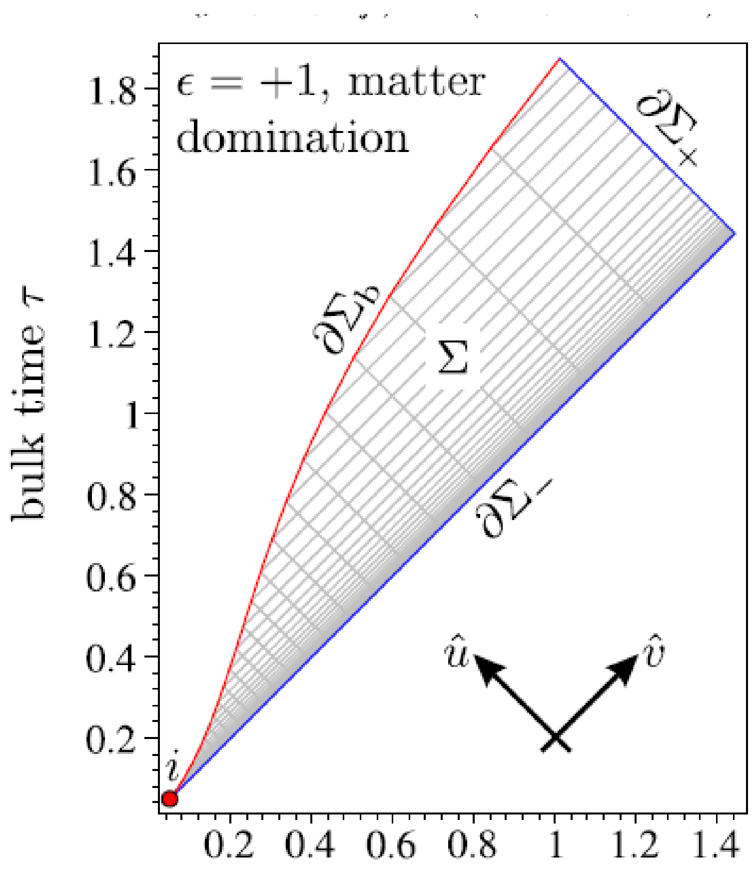
Inconsistent!

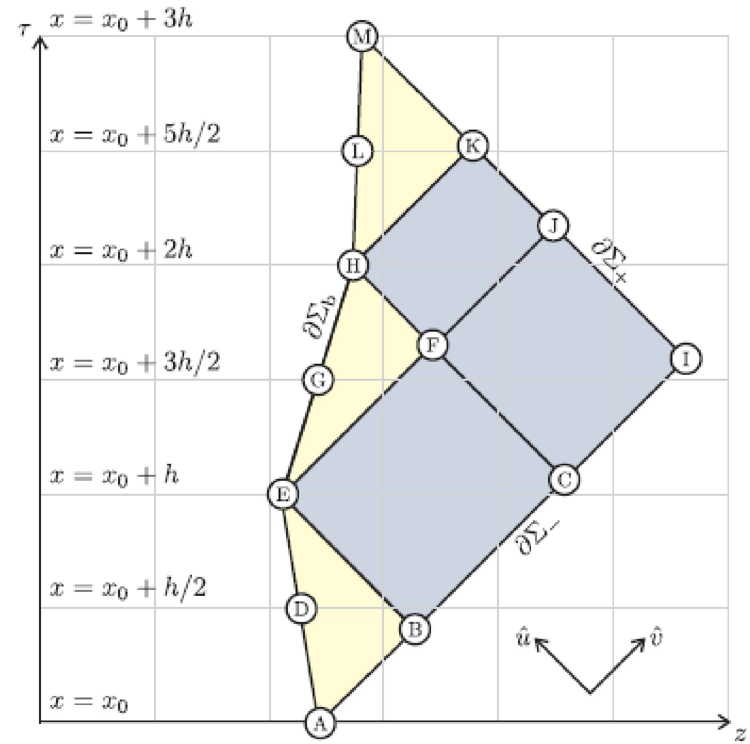
Gravity in DGP model



Full 5D linear analysis

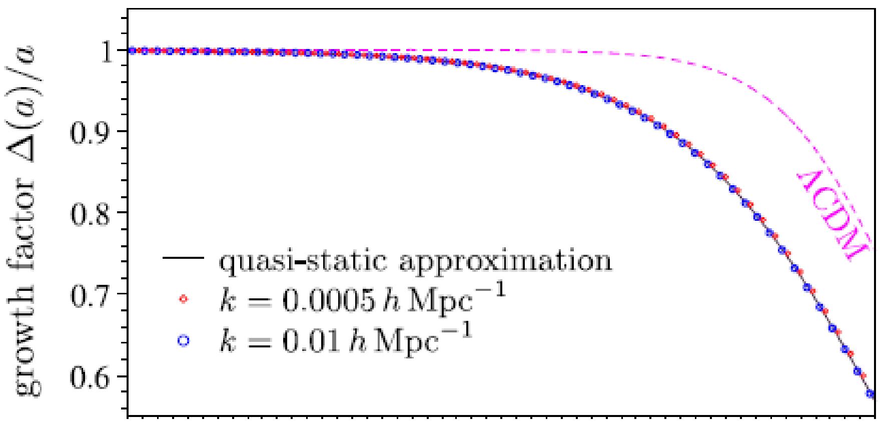
Cardoso, KK, Seahra and Silva, 0711.2563

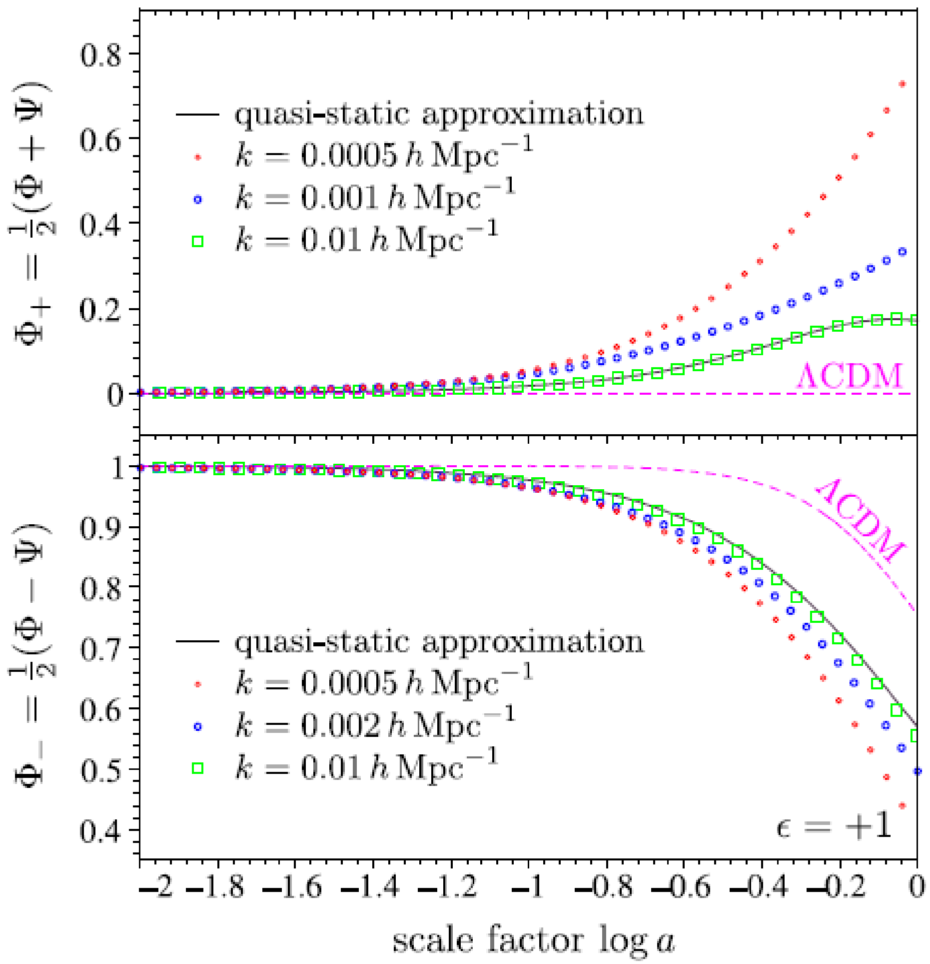




Solutions

$$ds_{\mathrm{b}}^2 = -(1+2\Psi)dt^2 + a^2(1+2\Phi)\delta_{ij}dx^i dx^j$$

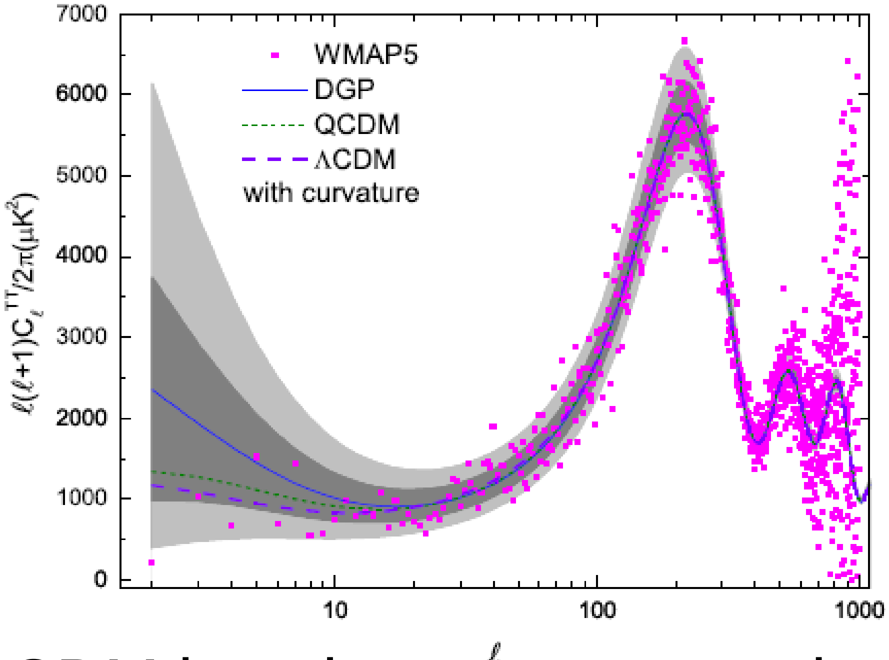




$$\Phi_+ = \frac{1}{2}(\Phi+\Psi) = 0$$ GR

$$\Phi_- = \frac{1}{2}(\Phi-\Psi)$$ CMB ISW

Enhancement of low multipoles



parameters	DGP	QCDM	ΛCDM
$100\Omega_b h^2$	2.38	2.36	2.27
$\Omega_c h^2$	0.0937	0.0960	0.107
$100\theta_s$	1.04	1.04	1.04
τ	0.0887	0.0914	0.0884
Ω_K	0.0189	0.0268	-0.00553
n_s	0.996	0.992	0.959
$\ln[10^{10} A_s]$	3.02	3.05	3.18
H_0	73.8	78.3	69.8
Ω_m	0.216	0.195	0.266
Ω_{r_c}	0.149	..	..
$-2\ln L$	2800.8	2787.2	2777.5

QCDM has the same expansion history as DGP

DGP is a poorer fit than LCDM at 5.3 σ level

inclusion of large scale CMB has 30% contribution to this conclusion

(Fang et.al. 0808.2208)

Quasi static solution

KK and Maartens,
JCAP [astro-ph/0511634]

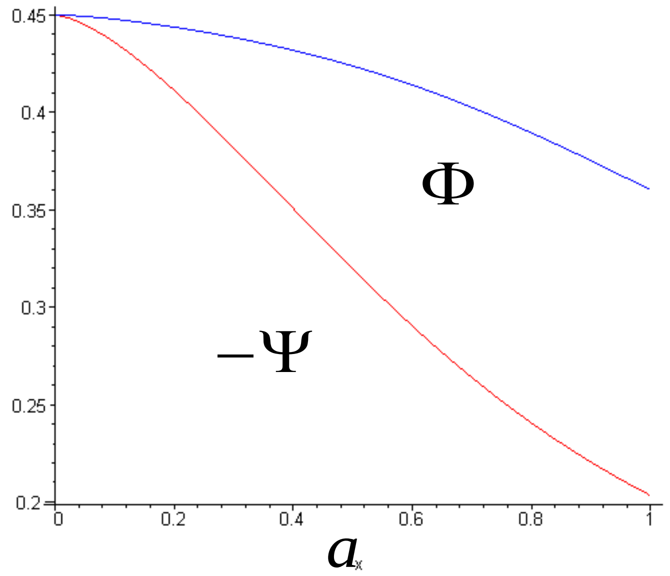
- Solutions for metric perturbations under horizon

$$ds^2 = -(1+2\Psi)dt^2 + a(t)^2(1+2\Phi)d\vec{x}^2$$

$$\frac{k^2}{a^2}\Phi = 4\pi G\left(1-\frac{1}{3\beta}\right)\rho\delta,$$

$$\frac{k^2}{a^2}\Psi = -4\pi G\left(1+\frac{1}{3\beta}\right)\rho\delta,$$

$$\beta = 1-2Hr_c\left(1+\frac{\dot{H}}{3H^2}\right)$$



Growth rate is determined by Ψ

O(1) modification to Newton's constant

(Lue, Sccoccimarro, Starkman)

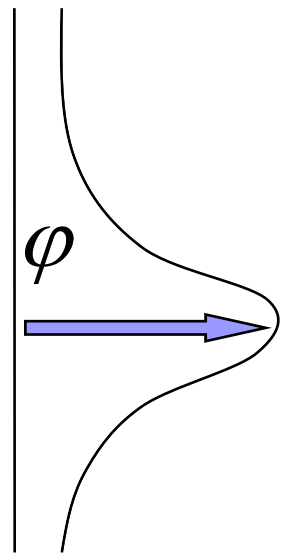
Non-linear evolution

Silva and KK

PRD [hep-th/0702169]

- Non-linearity of brane bending mode

$$ds^2 = -N^2\left(1+2\Psi\right)dt^2 + A^2\left(1+2\Phi\right)d\vec{x}^2 + (1+2G)dy^2$$
$$+2\,r_c\varphi_{,i}\,dydx^i$$



Solving bulk perturbations

imposing
- regularity condition in the bulk
- junction conditions on a brane

$$-\nabla^2\Phi = 4\pi G a^2\rho\delta + \frac{1}{2}\nabla^2\varphi, \qquad \Phi+\Psi = -\varphi \qquad \beta = 1-2Hr_c\left(1+\frac{\dot{H}}{3H^2}\right)$$

$$3\beta(t)\nabla^2\varphi + r_c^2\left\{\partial_j\left(\partial^j\varphi\,\nabla^2\varphi\right) - \partial_j\left(\partial^i\varphi\,\partial_i\partial^j\varphi\right)\right\} = 8\pi G a^2\rho\delta$$

Spherically symmetric solution

(Gruzinov; Middleton, Siopsis; Tanaka; Lue, Sccoccimarro, Starkman)

$$\frac{d\varphi}{dr} = \frac{r_g}{r^2}\Delta(r), \quad \Delta(r) = \frac{2}{3\beta}\left(\frac{r}{r_*}\right)^3\left(\sqrt{1+\left(\frac{r_*}{r}\right)^3}-1\right)$$

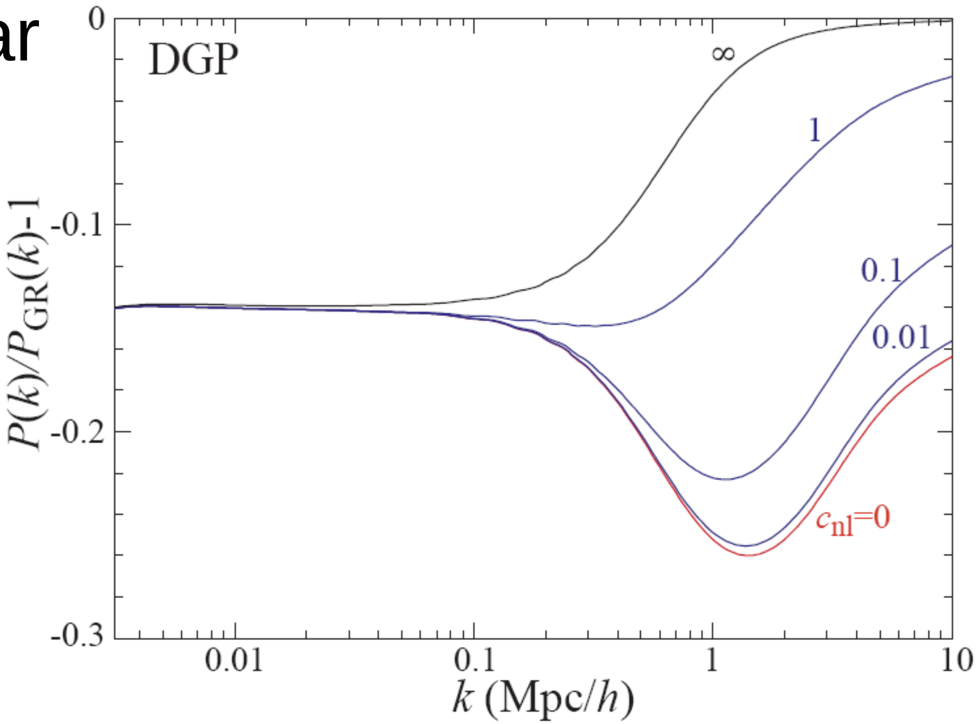
Vainstein radius $r_* = \left(\frac{8r_c^2 r_g}{9\beta^2}\right)^{\frac{1}{3}}, \quad r_g = 2G_4M$

4D Einstein ($r < r_*$)	4D BD ($r_* < r < r_c$)	5D ($r > r_c$)
$\Phi = \frac{r_g}{2r} + \frac{1}{\beta}\sqrt{\frac{\beta^2 r_g r}{2r_c^2}},$	$\Phi = \frac{r_g}{2r}\left(1-\frac{1}{3\beta}\right),$	
$\Psi = -\frac{r_g}{2r} + \frac{1}{\beta}\sqrt{\frac{\beta^2 r_g r}{2r_c^2}}$	$\Psi = -\frac{r_g}{2r}\left(1+\frac{1}{3\beta}\right)$	

Non-linear power spectrum

- Recovery of GR on small scales

we expect that non-linear power spectrum will go back to GR (=QCDM)



Fitting formula **(Sawicki, Hu)**

$$P(k) = \frac{P_0(k) + c_{nl}\Sigma^2(k)P_{QCDM}(k)}{1 + c_{nl}\Sigma^2(k)}$$

$$\Sigma^2(k) = \frac{k^3 P_{lin}(k)}{2\pi^2}$$

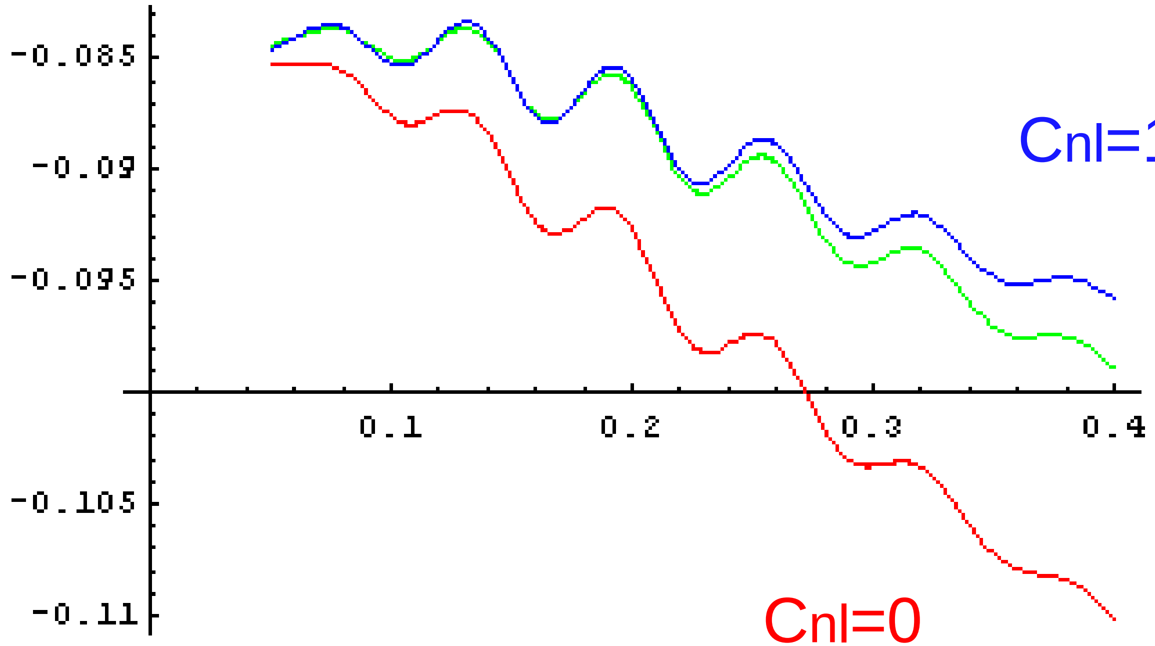
$P_0(k)$: non-linear power spectrum by neglecting the mechanism to recover GR

Quasi non-linear regime

(Hiramatus, Koyama and Taruya in preparation)

- 3rd order perturbation theory

$$\frac{P(k)}{P_{QCDM}(k)}-1$$



Within the validity regime of PT, we get $c_{nl}=1$

Is DGP model really a consistent theory for IR modification of gravity?

Strong coupling problem

- Covariant effective theory (Minkowski background)

$$S = \frac{1}{2}\left(\partial_\mu \varphi\right)^2 + \frac{1}{\Lambda}\left(\partial_\mu \varphi\right)^2 \left(\Box \varphi\right), \qquad \Lambda = \left(\frac{M_4}{r_c^2}\right)^{\frac{1}{3}}$$

We need quantum gravity below $\Lambda^{-1} = 1000\,\mathrm{km}$!

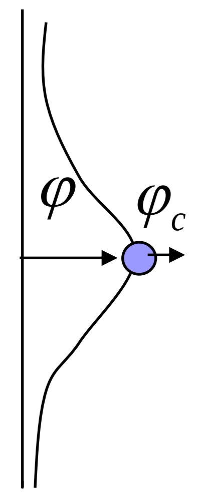
This is due to the fact φ disappears as $r_c \to \infty$

(Rubakov, Luty, Porrati and Rattazi)

- Loophole

Perturbations from Minkowski background do not make sense

(Dvali, Nicolis and Rattazi)



Ghost suppresses growth of structure

- Negative BD parameter

$$\omega = \frac{3}{2}(\beta - 1) \qquad \beta = 1 - 2Hr_c\left(1 + \frac{\dot{H}}{3H^2}\right)$$

In Einstein frame, kinetic term for the scalar $-\frac{3}{2}\beta$

if $\beta < 0$ the scalar becomes a ghost

cf. de Sitter spacetime

$$Hr_c \geq 1, \quad (\sigma \geq 0)$$

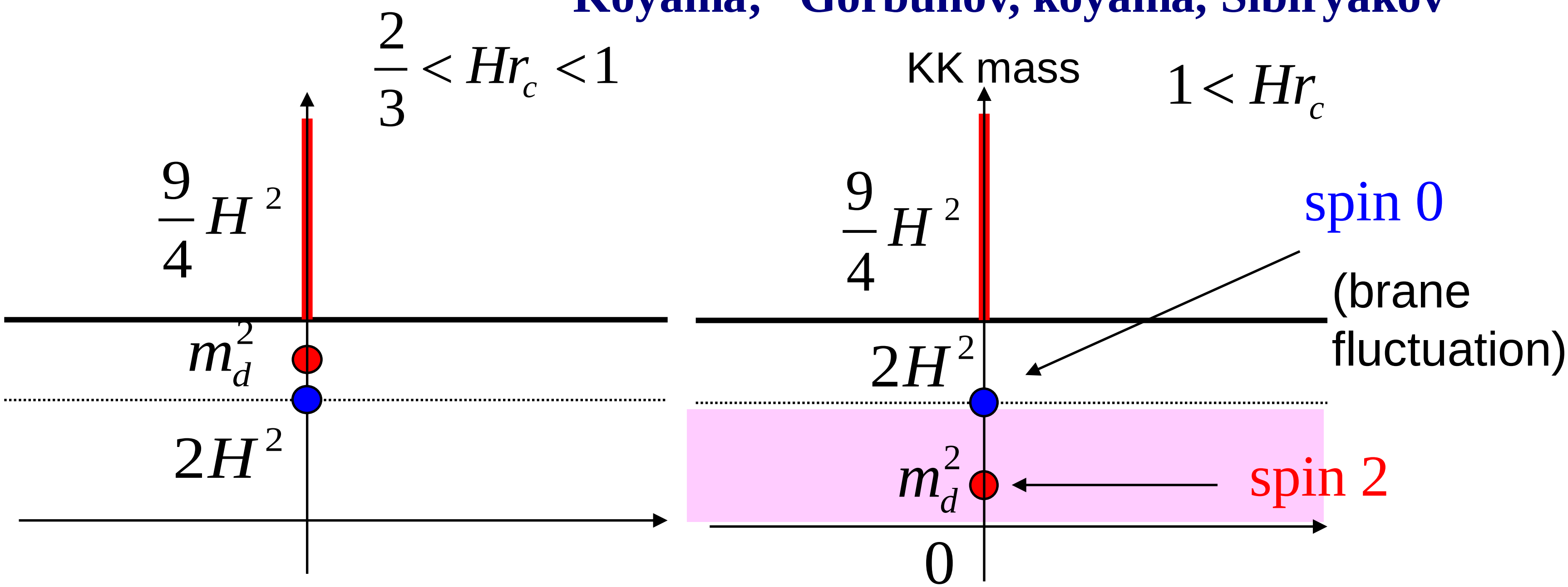
$$Hr_c < 1, \quad (\sigma < 0)$$

$$\beta < 0 \iff Hr_c > \frac{1}{2}$$

(Luty Porrati, Rattazzi; Nicolis and Rattazzi)

Ghost in de Sitter spacetime

Koyama; Gorbunov, koyama, Sibiryakov



Spin-2 helicity-0	non-ghost		ghost
Spin-0	non-ghost	ghost	non-ghost
		$\frac{1}{2}$	1 Hr_c

(Charmousis et.al)

DGP ghost **(Izumi and Tanaka)**

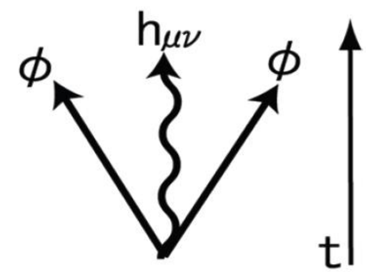
- Ghost is helicity-0 mode in spin-2 gravity

  If we want to avoid negative norm state we should treat helicity-0 mode separately

  ⇨ already breaks de Sitter invariance

  Cut off scale could come from the strong coupled scale

$$r_{strong} \approx \left(l_p \, r_c^2 \right)^{\frac{1}{3}} \sim 1000km$$



Then spontaneous pair production of particle could be suppressed

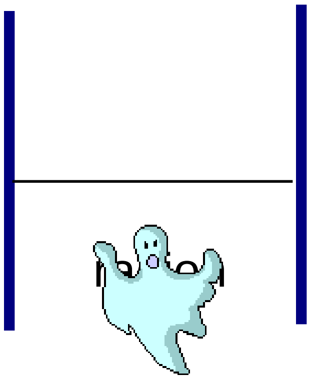
$$\rho \approx \frac{H^2}{M^2_{4pl}} \Lambda^4$$

Modifications of the model

- 5D models (Izumi, Koyama, Tanaka)

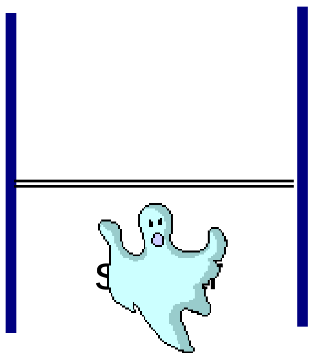
Two-brane model

spin-2 ghost is removed but radion become a ghost



Stabilisation

radion ghost is removed but the scalar field becomes a ghost



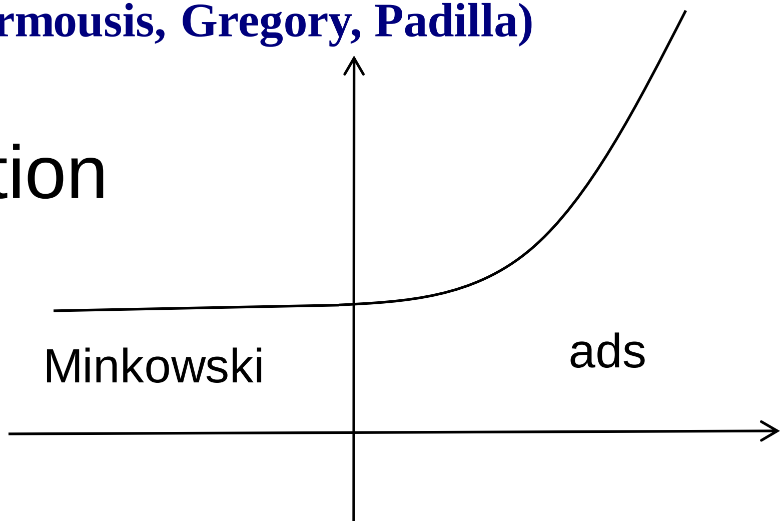
(Charmousis, Gregory, Padilla)

- Stealth acceleration

asymmetric brane configuration

Minkowski bulk + AdS bulk

induced gravity on a brane



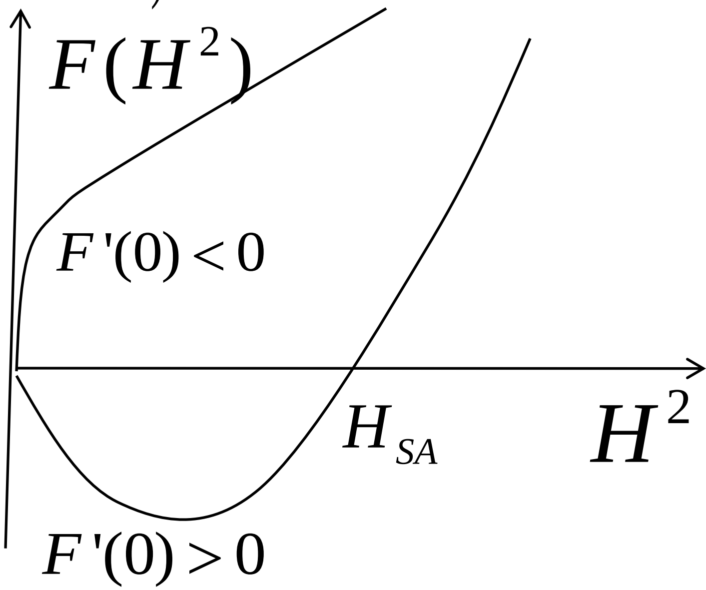
$$\rho = F(H^2)$$

$$F(H^2) = 6M_4^2 H^2 + 6M_L^3 H - 6M_R^3\left(\sqrt{H^2 + k^2} - k\right)$$

If $F'(0) \geq 0$ no self-acceleration

Minkowski spacetime has

no ghost only if $F'(0) \to \infty$!

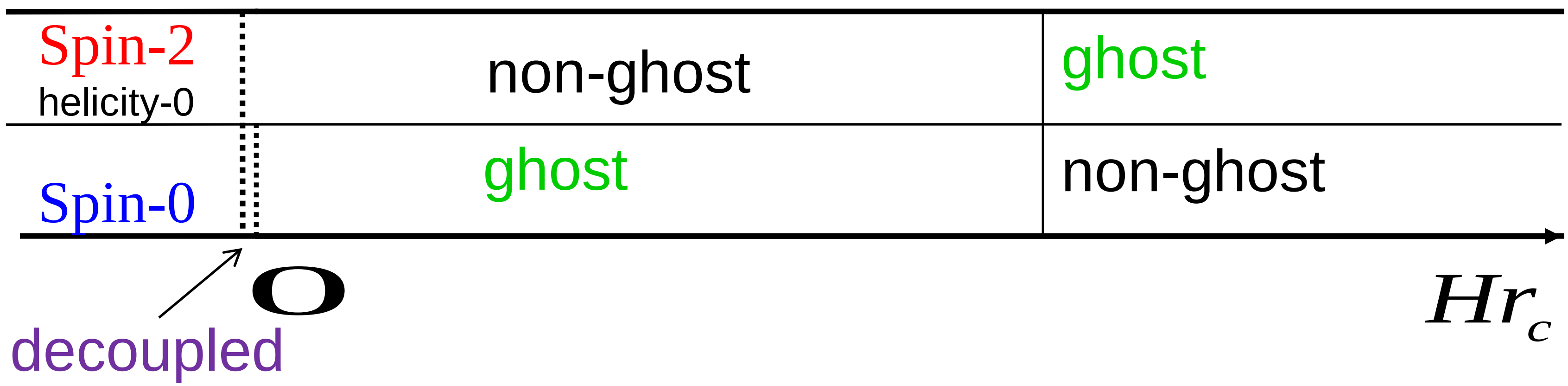


Stealth acceleration

no acceleration without matter but the modified Freidman equation yields acceleration even with ordinary matter

However, in the expanding universe...

(Silva, Koyama, Padilla)

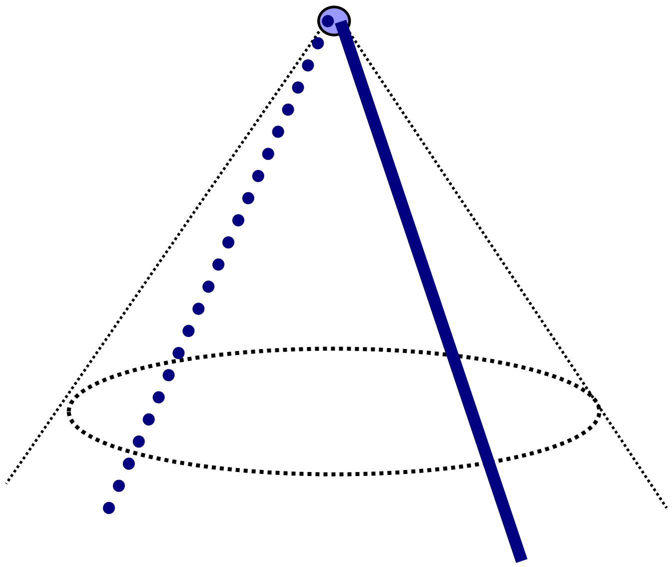


Although the ghost is decoupled in Minkowski spacetime, we have the same problem as in the DGP in early universe

- 6D extension

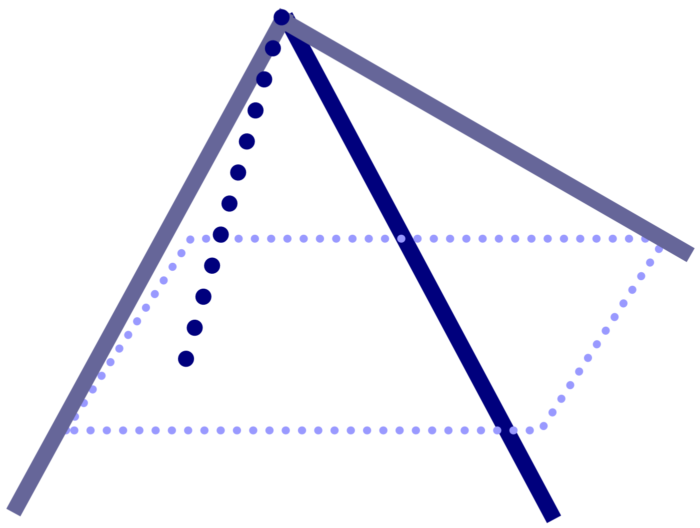
cascading

de Rham et.al.



intersecting

Corradini, Koyama Tasinato



6D gravity introduce an additional scalar graviton

It is still needed to see this improves the situation

Toward MG as an alternative to DE

- **Model building**
  - Construct consistent MG models

    new models wanted!
  - address fundamental problem

    cf. C.C. problem, fine-tuning, stability
- **Observational predictions**
  - Three regime of modified gravity models

    largest scales / linear scale / non-linear scales
- **Observational tests**
  - Combine geometrical and structure formation tests on various scales
  - Model independent tests