Big-bang nucleosyntheis and a hint to solve problems in astrophysics, cosmology and particle physics

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- Solving lithium problem

Brief review of Big-bang nucleosynthesis (BBN)

Thermal history of the Universe							
Big ba	ing		cf) 1 GeV ~ 10 ¹³ K				
time "t	emperature	"					
10^{-114} sec -	10 ¹⁸ GeV	Planck scale	Inflation and Reheating				
10^{-38} sec	10 ¹⁶ GeV	GUT phase transition?					
10^{-11} sec -	10^2 GeV	Electroweak phase transit	ion Baryogenesis?				
10^{-5} sec -	200 MeV	QCD phase transition					
0.1 sec	3 MeV	Neutrino decoupling	Big-Bang				
1 sec	0.5 MeV	Electron-positron annihila	ation Nucleosynthesis				
10^{10} sec -	1 eV	Matter-radiation equality	(BBN)				
10^{12} sec -	0.1 eV	Photon decoupling (CMB	5)				
13.7_{17} Gyr (~10 ¹⁷ sec)	10^{-4} eV	Present					

Thermal history around BBN Epoch cf) 100 MeV ~ 10 ¹² K								
10 ⁻⁴ sec	100 MeV	Muon annihilation $(\mu^+ + \mu^- \rightarrow 2\gamma)$						
	\approx							
0.1 sec		Neutrino decoupling $(v_e + e^- \neq v_e + e^-)$						
1 sec	- 1 MeV	n/p freezeout $(n + v_e \neq p + e^-)$						
	0.5 MeV	Electron-positron annihilation						
10^2 sec	- 0.1 MeV	Beginning of BBN $(e^+ + e^- \rightarrow 2\gamma)$						
10^3 sec	- 0.07 MeV	End of (Standard) BBN						

Scenario of BBN 1) T > 1 MeV (t < 1 sec)Radiation γ, e^{\pm}, v n, p

Weak interaction is in equilibrium

$$n + e^+ \leftrightarrow p + \nu_e$$

$$\frac{n_n}{n_p} = Exp\left[-\frac{Q}{T}\right]$$

 $(\mathbf{Q} \equiv m_n - m_p \sim 1.29 \text{ MeV})$

2) $T \sim 1 \text{ MeV} (t \sim 1 \text{ sec})$ cf) 1 MeV ~ 10¹⁰K

Feezeout of weak interaction

•Weak interaction rate

•Hubble expansion rate

 $\Gamma_{n \leftrightarrow p} \sim \sigma_{n \leftrightarrow p} n_e \sim G_F^2 T^5$

$$H = \frac{\dot{a}(t)}{a(t)} \sim T^2 / M_{pl}$$

$$\frac{\Gamma}{H} \approx \left(\frac{T}{0.8 \text{ MeV}}\right)^2$$

 $\Gamma < H$ $(T < 0.8 \text{ MeV} \equiv T_f)$ (n_n/n_p) is fixed

$$\left(\frac{n_n}{n_p}\right)_{\text{freezeout}} \approx Exp\left[-\frac{Q}{T_f}\right]$$

He4 mass fraction



3) $T \sim 0.1 \text{ MeV} (t \sim 100 \text{ sec})$ cf) 0.1 MeV ~ 10^{9} K $p + n \rightarrow D + \gamma$ $T \ll B_D = 2.2 \text{ MeV}$ 4) T < 0.1 MeV (t > 100 sec) $n_D / n_H \sim 16.3 (T / m_N)^{3/2} \eta \exp[B_D / T] > 0.01$ $D + D \rightarrow T + p$, ³He+n $T + D ({}^{3}\text{He} + D) \rightarrow {}^{4}\text{He} + n ({}^{4}\text{He} + p)$ A little *D* and ³He are left as cold ashes

There is no stable nuclei for A=5,8. Mass 7 nuclei are produced a little.

⁴He +
$$T \rightarrow {}^{7}Li + \gamma$$

⁴He + ${}^{3}He \rightarrow {}^{7}Be + \gamma$
 $\downarrow {}^{7}Be + e^{-} \rightarrow {}^{7}Li + \nu_{e}$
⁴He + $D \rightarrow {}^{6}Li + \gamma$

Time evolution of light elements



Observational light element abundances 1) ⁴He

•Observing the recombination line in Metal poor extragalactic H II region, or blue compact galaxy

•Extrapolating them into zero metalicity



Fukugita, Kawasaki (06)

 $Y_{p} = 0.250 \pm 0.004$ Fukugita, Kawasaki (06) $Y_{p} = 0.2474 \pm 0.0028$ Peimbert, Luridiana, Peimbert (07) $Y_{p} = 0.2516 \pm 0.0011$

Izotov, Thuan, Stasinska (07)



Observed in high redshift QSO absorption system







Burles and Tytler (1997)

$D/H = (2.82 \pm 0.26) \times 10^{-5}$

O'Meara et al.(2006)

2) Lithium 7

•Observing metal poor halo stars in Pop II

•Abundance does not depend on metalicity so much for

"Spite's plateau"



$$Log_{10}(^{7}Li/H) = -9.63 \pm 0.06$$

Melendez and Ramirez(2004)

Observational Light Element						
He4	$Y_p = 0.2516 \pm 0.004$	Fukugita, Kawasaki (2006) Peimbert,Lridiana, Peimbert(2007) Izotov,Thuan, Stasinska (2007)				
۵.	$D/H = (2.82 \pm 0.26) \times 10^{-5}$	O'Meara et al. (2006)				
• Li7	$\log_{10}(^{7}\text{Li/H}) = -9.63 \pm 0.06 (\pm 0.3)$) _{syst.} Melendez,Ramirez(2004)				
Li6	⁶ Li / ⁷ Li < 0.046 ± 0.022 (±0.084) _{sys} Asplund et al(2006)				
• He3	³ He/D < 0.83 + 0.27	Geiss and Gloeckler (2003)				

SBBN



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Brief review of Dark Matter (DM) in Supersymmetry (SUSY)



Realistic candidates of particle dark matter in SUSY/SUGRA

- Neutralino χ (Bino, wino, or higgsinos) Most famous Lightest Supersymmetric Particle (LSP) with $m_{\chi} \sim 100 GeV$ (appears even in global SUSY)
- Gravitino ψ_{μ} super partner of graviton with spin 3/2 and $m_{3/2} \leq 100$ GeV (massive only in SUGRA (local SUSY))

Introduction to SUSY Supersymmetry (SUSY) Solving "Hierarchy Problem" Realizing "Coupling constant unification in GUT" Fermion Boson quark squark lepton + slepton



• GUT-scale

$$M_X \approx 10^{14} - 10^{15} \text{GeV}$$

• Weak-scale
• $M_X = 10^2 + 10^3 \text{GeV}$

$$M_{W} \approx 10^{2} - 10^{3} GeV$$

Higgs mass

$$m_{\varphi 0}^{2} = \frac{d^{2} V_{\varphi}}{d \varphi^{2}} \approx \lambda \upsilon^{2} \approx \mathcal{O}(\mathcal{M}_{W}^{2})$$

where Higgs's potential

$$V_{\varphi} = \lambda \left(\varphi^{\dagger} \varphi - \upsilon^{2} / 2 \right)^{2}$$



c.f) Masses of fermions and vector bosons $m_{\psi} \sim h_{\psi} \langle \varphi \rangle, \ m_{Z} \sim g \langle \varphi \rangle$

Radiative correction to Higgs mass in Quantum Field Theory



$$\delta m_{\varphi}^2 \sim (10^{15} \text{ GeV})^2$$
 ?

How can we resolve the problem?

Weak scale in the tree level, $m_{\varphi 0}^2 \sim (10^2 \text{GeV})^2$

In total,

$$\delta m_{\varphi}^{2} \sim (10^{15} \text{GeV})^{2}$$
$$m_{\varphi}^{2} \sim + \delta m^{2} \sim (10^{15} \text{GeV})^{2}$$

To retain the hierarchy, we require an <u>accidental</u> cancellation,

$$m_{\varphi 0}^{2} + (\delta m_{\varphi 1}^{2}) + (\delta m_{\varphi 2}^{2}) + (\delta m_{\varphi 3}^{2}) \dots \sim (10^{2} \text{GeV})^{2}$$
?

 $|\mathcal{O}(10^{15} \text{GeV})|^{2}$

GDP in USA (2002)?

\$ 10,110,087,734,958.95 -) \$ 10,110,087,734,957.70

\$ 1.25

Solution in SUSY

In exact SUSY, the quadratic divergence is canceled by both boson and fermion loops.



Even if SUSY,

$$\delta m_{\varphi}^{2} \sim \frac{1}{(4\pi)^{2}} h_{t}^{2} m_{\tilde{t}}^{2} \ln\left(\frac{\Lambda^{2}}{m_{\tilde{t}}^{2}}\right)$$

We don't need a fine tuning when

 $m_{\tilde{t}}^2 \sim m_{\tilde{b}}^2 \sim \dots \sim O(M_W^2)$

SUSY GUT

The coupling constants are unified at

 $M_{\chi} \approx 10^{16} GeV$

A lot of new particles ,which do not obey the asymptotic free, appear at





MSSM

 Minimal extension of Standard Model to supersymmetry including two Higgs doublets

 $W_{MSSM} = -\overline{u}y_u QH_u + dy_d QH_d + \overline{e}y_e LH_d$ $\overline{d}y_d QH_u^* \text{ because of holomorphism in super pot.}$ $H_u = \begin{pmatrix} h_u^+ \\ h_u^0 \end{pmatrix} \qquad H_d = \begin{pmatrix} h_d^0 \\ h_d^- \end{pmatrix}$

105 masses, phases and mixing angles!!!



Constrained MSSM

Simplified into only five parameters from 105

- (1) Common scalar mass at GUT scale: m_0
- 2 Unified gaugino (fermion) mass at GUT scale: $m_{1/2}$
- 3 Ratio of Higgs vacuum expectation values:

 $\tan\beta \equiv \frac{\left\langle \mathcal{H}_{u}^{0}\right\rangle}{\left\langle \mathcal{H}_{d}^{0}\right\rangle}$

- 4 Higgs/higgsino mass parameter (or its signature): μ
- (5) tri-linear coupling A_0

Super particles in CMSSM

		3.7	~ .	T			
		Names	Spin	P_R	Gauge Eigenstates	Mass Eigenstates	
		Higgs bosons	0	+1	$H_{u}^{0} H_{d}^{0} H_{u}^{+} H_{d}^{-}$	$h^0 H^0 A^0 H^{\pm}$	
					$\widetilde{u}_L \ \widetilde{u}_R \ \widetilde{d}_L \ \widetilde{d}_R$	(same)	
		squarks	0	-1	$\widetilde{s}_L \ \widetilde{s}_R \ \widetilde{c}_L \ \widetilde{c}_R$	(same)	
					$\widetilde{t}_L \ \widetilde{t}_R \ \widetilde{b}_L \ \widetilde{b}_R$	$\widetilde{t_1}\widetilde{t_2} \widetilde{b}_1 \widetilde{b}_2$ S	top
					$\widetilde{e}_L \ \widetilde{e}_R \ \widetilde{\nu}_e$	(same)	
		sleptons	0	-1	$\widetilde{\mu}_L \ \widetilde{\mu}_R \ \widetilde{ u}_\mu$ S	tau (safie)eutri	no
10	V	vino hiaas	inos		$\widetilde{\tau}_L \ \widetilde{\tau}_R \ \widetilde{\nu}_{\tau}$	$\widetilde{\tau_1}\widetilde{\tau_2}\widetilde{\nu_{\tau}}$	
		neutralinos	1/2	-1	$\widetilde{B}^0 \widetilde{W}^0 \widetilde{H}^0_u \widetilde{H}^0_d$	$\widetilde{N}_1 \ \widetilde{N}_2 \ \widetilde{N}_3 \ \widetilde{N}_4$	
		$\operatorname{charginos}$	1/2	-1	\widetilde{W}^{\pm} \widetilde{H}^+_u \widetilde{H}^d	\widetilde{C}_1^{\pm} \widetilde{C}_2^{\pm}	
		gluino	1/2	-1	\widetilde{g}	(same)	
		goldstino (gravitino)	$\frac{1/2}{(3/2)}$	-1	\widetilde{G}	(same)	

bi

Running of Renormalization Group (RG) Equation in CMSSM



Negative Higgs mass term

Mass spectrum in CMSSM



Lightest SUSY particle (LSP)

• R-parity conservation i)Decay $\tilde{\tau} \rightarrow \chi + \tau$ (-1) (-1)× (+1) ii) Pair annihilation/production

$$\begin{array}{l} f+f \leftrightarrow \chi + \chi \\ (+1) \times (+1) & (-1) \times (-1) \end{array}$$



 Ω does not depend on m_y so much

Predicting TeV Physics!!!

 $\langle \sigma v \rangle = 3 \times 10^{-26} \, cm^3 \, / \, s$

LSP (LOSP) in CMSSM

Neutralino or Scalar tau lepton (Stau) is the Lightest Ordinary SUSY Particle (LOSP)



Ellis,Olive,Santoso,Spanos(03)

Supergravitiy (SUGRA)

- Local theory of SUSY (predicting gravitino)
- Models of supersymmetry breaking (gravitino mass production by eating goldstino which appears in spontaneous symmetry breaking)
- Including general relativity (Unifying space-time symmetry with local SUSY transformation)

<u>SUSY Breaking Models</u>

Gravity mediated SUSY breaking model

Observable sector quark, squark, ... Only through gravity



Hidden sector SUSY $F \sim 10^{20-21} GeV^2$

Masses of squarks and sleptons

 $m_{\tilde{q}}, m_{\ell} = F / M_{p/} = 10^{2} - 10^{3} \text{ GeV}$ (F = 10²⁰ - 10²¹ GeV) • Gravitino mass $m_{3/2} = F / M_{p/} = 10^{2} - 10^{3} \text{ GeV}$

<u>SUSY Breaking Models II</u>

Gauge-mediated SUSY breaking model


Advanced topics

Lithium Problem

If we adopted no systematic errors for observational data of 6Li and 7Li, the BBN theory does not agree with observation of Li abundances.



Lithium 7

a factor of two or three smaller !!!

• Expected that there is little depletion in stars.



⁷Li/H = $2.19^{+2.2}_{-1.1} \times 10^{-10}$ (1 σ) $\log(^{7}Li/H) = -9.63 \pm 0.06$ (1 σ) Bonifacio et al.(2002) Melendez,Ramirez(2004) ⁷Li/H = $1.23^{+0.68}_{-0.32} \times 10^{-10}$ (1 σ) $\log(^{7}Li/H) = -9.90 \pm 0.09$ (1 σ)

Ryan et al.(2000)

Lithium 6

Asplund et al.(2006)

•Observed in metal poor halo stars in Pop II

●⁶Li plateau?



6
Li / 7 Li = 0.022 – 0.090

 7 Li/H \approx (1.1–1.5)×10⁻¹⁰ still disagrees with SBBN

Astrophysically, factor-of-two depletion of Li7 needs a factor of O(10) Li6 depletion (Pinsonneault et al '02) We need more primordial Li6?

Doppler broadening

Cold ISM

Knauth,Federman,La mbert (2006)





LP815-43

Asplund et al.(2006)

Astrophysical uncertainties in Li7

Diffusion and convection
 Korn et al (2006)



Destruction of Li7 in inner zone of stars

There is a trend of Iron and Li7 as a function of T_{eff}



Astrophysical uncertainties in Li6

Cayrel, Steffen, Bonifacio, Ludwig and Caffau (2008)

• Asymmetries of the absorption line mimicked by convective motion





My attitude towards Li problem

It might be premature to accept that these observational values are purely primordial

 Adding large systematic errors by hand and constraining (non-)standard cosmological scenarios

• Inventing a new cosmological/particle-physics model to solve Li problem

Constraints on long-lived SUSY particles from BBN

• We add large systematic errors into observational Li7 and Li6 abundacnes

 $\log_{10}(^{7}\text{Li}/\text{H}) = -9.63 \pm 0.06 (\pm 0.3)_{\text{syst.}}$

Melendez,Ramirez(2004)

 $^{6}Li / ^{7}Li < 0.046 \pm 0.022 (\pm 0.084)_{sys}$

Asplund et al(2006)



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Sato and Kobayashi (1977), Lindley (1984,1985), Khlopov and Linde (1984)
Ellis, Kim, Nanopoulos, (1984); Ellis, Nanopoulos, Sarkar (1985)
Kawasaki and Sato (1987)
Reno and Seckel (1988), Dimopoulos, Esmailzadeh, Hall, Starkman (1988)
Kawasaki, Moroi (1994), Sigl et al (95), Holtmann et al (97)
Jedamzik (2000), Kawasaki, Kohri, Moroi (2001), Kohri(2001), Cyburt, Ellis,
Fields, Olive (2003)
Kawasaki, Kohri, Moroi(04), Jedamzik (06)
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Radiative decay mode



1) Electro-magnetic cascade

$$\gamma + \gamma_{\rm BG} \to e^+ + e^-$$

$$\gamma + e^-_{\rm BG} \to \gamma + e^-, \quad e^- + \gamma_{\rm BG} \to e^- + \gamma$$

$$\gamma + \gamma_{\rm BG} \to \gamma + \gamma$$

2) many soft photons are produced

3) Photo-dissociation of light elements

<u>Hadronic decay mode</u>

Reno, Seckel (1988)

S. Dimopoulos et al.(1989)



Two hadron jets with $E_{jet} = m_{\chi}/3$



(I) Early stage of BBN (T > 0.1MeV)

Reno and Seckel (1988) Kohri (2001) Extraordinary inter-conversion reactions between n and p cf) $n + \pi^+ \rightarrow p + \pi^0$ $p + \pi^- \rightarrow n + \pi^0$



(II) Late stage of BBN (T < 0.1MeV)

Hadronic showers and "Hadro-dissociation"

S. Dimopoulos et al. (1988) Kawasaki, Kohri, Moroi (2004)



Non-thermal Li, Be Production by energetic nucleons or photons

dimopoulos et al (1989) Jedamzik (2000)

(1) T(He3) - He4 collision $T + {}^{4}\text{He} \rightarrow {}^{6}\text{Li} + n \quad [8.4 \text{ MeV}]$ ${}^{3}\text{He} + {}^{4}\text{He} \rightarrow {}^{6}\text{Li} + p \quad [7.0 \text{ MeV}]$

2 He4 - He4 collision

 $^{4}\text{He} + {}^{4}\text{He} \rightarrow {}^{6}\text{Li}, {}^{7}\text{Li}, {}^{7}\text{Be} + \dots$

Massive particle X Upper bounds on m_xY_x in both photodissociation and "hadrodissociation" scenario Kawasaki, Kohri, Moroi (04)



Neutralino (bino) LSP and gravitino "NLSP"

Upper bound on reheating temperature

Kawasaki, Kohri, Moroi, Yotusyanagi (08)



$$Y_{x} \equiv n_{x} / s$$

Ruwusuki, Koni I, Moroi, Yorusyunugi (00)

$$T_R \approx 10^9 \text{GeV}(Y_{3/2}/10^{-12})$$

$$\tau \sim m_{pl}^2 / m_{3/2}^3$$

	Case 1
$m_{1/2}$	$300 {\rm GeV}$
m_0	$141 \mathrm{GeV}$
A_0	0
aneta	30
μ_H	$389 {\rm GeV}$
$m_{\chi^0_1}$	$117 { m GeV}$
$\Omega_{ m LSP}^{(m thermal)}h^2$	0.111

Neutralino (bino) NLSP and gravitino LSP

<u>Gravitino LSP and thermally porduced</u> <u>neutralino (Bino) "NLSP" scenario</u>

Lifetime

$$\tau \sim m_{3/2}^2 m_{pl}^2 / m_{NLSP}^5$$

Relic abundance

$$Y_{\tilde{B}} = 4 \times 10^{-12} \times \left(\frac{m_{\tilde{B}}}{100 \text{ GeV}}\right)$$
 : bulk

Feng, Su, and Takayama (03) Steffen (06) Kawasaki, Kohri, Moroi, Yotsuyanagi (08)

No allowed region for DM density

Sneutrino NLSP and gravitino LSP scenario

Stable (left-handed) sneutrino was excluded by the direct detection experiments because of its large cross section directly-coupled with W/Z bosons.

(left-handed) sneutrino should be unstable

<u>Gravitino LSP and thermally porduced</u> <u>sneutrino NLSP scenario</u>

Relic abundance

Kawasaki, Kohri, Moroi, Yotsuyanagi (08) Ellis, Olive, Santoso (08)

 $Y_{\tilde{\nu}} \simeq 2 \times 10^{-14} \times \left(\frac{m_{\tilde{\nu}}}{100 \text{ GeV}}\right)$

No allowed region for DM density with 100GeV sneutrinos

Stau NLSP and gravitino LSP scenario

Stable stau with weak-scale mass (<100TeV) was excluded by the experiments of ocean water

NLSP stau should be unstable

Bound-state effect (see next)

CHArged Massive Particle (CHAMP)

Kohri and Takayama, hep-ph/0605243 See also literature, Cahn-Glashow ('81)

Candidates of long-lived CHAMP in modern cosmology stau, stop ...

> "CHAMP recombination" with light elemets $T_c \sim E_{bin}/40 \sim 10 \text{ keV}$ (E_{bin} $\sim \alpha^2 \text{ m}_i \sim 100 \text{ keV}$)

See also the standard recombination between electron and proton, ($T_c \sim E_{bin}/40 \sim 0.1 eV$, $E_{bin} \sim \alpha^2 m_e \sim 13.6 eV$)) CHAMP captured-nuclei, e.g., (C,⁴He) changes the nuclear reaction rates dramatically in BBN

Pospelov's effect

Pospelov (2006), hep-ph/0605215

 CHAMP bound state with ⁴He enhances the rate

$$D + (^{4}He, C^{-}) \rightarrow ^{6}Li + C^{-}$$

Enhancement of cross section

~ $(\lambda_{\gamma} / a_{Bohr})^5$ ~ $(30)^5$ ~ 10^{7-8}

Confirmed by Hamaguchi etal (07), hep-ph/0702274

Stau NLSP and gravitino LSP Scenario in gauge mediation

Kawasaki, Kohri, Moroi PLB 649 (07) 436

Relic abundance

$$Y_{\tilde{\tau}} \simeq 7 \times 10^{-14} \times \left(\frac{m_{\tilde{\tau}}}{100 \text{ GeV}}\right)$$

Residual DM annihilation even at around BBN epoch

• To fit the PAMELA and ATIC2 positron signals and EGRET gamma-ray anomaly,

 $\langle \sigma v \rangle \sim 10^{-24} - 10^{-23} cm^3 / s$

- At least the annihilation must emit charged leptons Electromagnetic cascade shower is induced
- The annihilation might also emit hadrons Hadronic cascade shower is induced

Hadron emission by residual DM annihilation into WW in BBN epoch

Hisano et al in preparation

Solving Li problem in new particle physics models

• We do not adopt systematic uncertainties of observational Li7 and Li6 abundances

Reduction of ⁷Li and production of ⁶Li Jedamzik (04), Cumberbatch et al (08)

- Copious neutrons and tritiums are produced in hadronic shower process in decay/annihilation
- Reducing Be7 through

Jedazmik ('04)

⁷Be(**n**,p)⁷Li(p, ⁴He)⁴He

(⁷Li is produced later by ⁷Be + $e^- \rightarrow {}^7Li + v_e$) • Tritium scatters off the background He4 and produces Li6 Dimopoulos et al ('89) Kawasaki, Kohri, Moroi ('04)

 $T+4He \rightarrow {}^{6}Li+n$

Stop NLSP and gravitino LSP

Kohri and Santoso (08)

- Stop can be NLSP in Non-universal Higgs masses (NUHM)
- Stop is confined into "messino" after QCD phase transition
- Second annihilation of stop occurs just after QCD phase transition through strong interaction
- Stop number density is highly suppressed, but it is appropriate to solve the Li problem

 $m_{f}n_{f}/s \sim 10^{-14} GeV - 10^{-13} GeV$

Stop NLSP and gravitino LSP Kohri and Santoso <u>arXiv:0811.1119v1</u> [hep-ph]

Residual annihilation of wino-like LSP

Hisano, Kawasaki, Kohri, Nakayama(08)

Conclusion

- BBN is a strong tool to investigate decays or annihilations of long-lived SUSY particles.
- In neutralino LSP and unstable gravitino scenario, the constraint on reheating temperature after primordial inflation is

 $T_R \le 3 \times 10^5 \text{GeV} - 10^7 \text{ GeV}$ (for $m_{3/2} = 100 \text{ GeV} - 1\text{TeV}$)

 In the gravitino LSP scenario, thermal-relic NLSP fails to produce DM density for NLSP masses (100GeV - 1TeV). We need thermal or nonthermal production of LSP gravitino through the decay of Inflaton, moduli, etc.
 See Moroi, Murayama, Yamaguchi (93) for thermal production, and Endo, Takahashi, Yanagida (07) for non-thermal production of LSP gravitino